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CALCULATION OF INFORMATION CONTENT IN AXIOMATIC DESIGN

Gwang-Sub Shin
g.shin@samsung.com
Mechanical Design and Production Engineering, Hanyang University
Haengdang-1-Dong 17, Seongdong-Gu, Seoul 133-791, Korea

Jeong-Wook Yi
yijwook@ihanyang.ac.kr
Mechanical Design and Production Engineering, Hanyang University
Haengdang-1-Dong 17, Seongdong-Gu, Seoul 133-791, Korea

Sang-Il Yi
mechanic99@ihanyang.ac.kr
Mechanical Design and Production Engineering, Hanyang University
Haengdang-1-Dong 17, Seongdong-Gu, Seoul 133-791, Korea

Yong-Deok Kwon
kyd090@orgio.net
Mechanical Design and Production Engineering, Hanyang University
Haengdang-1-Dong 17, Seongdong-Gu, Seoul 133-791, Korea

Gyung-Jin Park
gjpark@hanyang.ac.kr
Department of Mechanical Engineering, Hanyang University
Sa-1-Dong 1271, Ansan City 425-791, Gyeonggi-Do, Korea

ABSTRACT

Axiomatic design offers a scientific base for design in an efficient way. It is well known that it has two axioms: the Independence Axiom and the Information Axiom. Many applications of the Independence Axiom have been published, however, the Information Axiom has been mainly applied to 1FR (functional requirement)-1DP (design parameter) problems except for a few case studies. This research presents various methods for calculation of information content. Generally, the information content is evaluated by the probability of success. The probability of success is calculated in two ranges: the FR range and the DP range. In the FR range, the graphical method is utilized with uniform distribution of the DP. In the DP range, the integration method is employed. It is noted that any distribution function of the DP can be accommodated in the integration method. The developed method can be applied to a decoupled design with multiple FRs and DPs. The method is extended to a coupled design and a design with a hierarchical structure of axiomatic design.

Keywords: Axiomatic design, Information Axiom, Information content

1 INTRODUCTION

Axiomatic design is one of the useful design methodologies. It consists of two axioms for general design [Suh, 1990, 2001]. The first axiom is the Independence Axiom. It states that the independence of functional requirements (FRs) must always be maintained, where FRs are defined as the minimum set of independent requirements that characterize the design goals [Do and Park, 1996, 2001; Lee and Park, 2000; Gebala and Suh, 1999]. The second axiom is the Information Axiom, and it states that among those designs that satisfy the Independence Axiom, the design with the smallest information content is the best design [Albano, 1993; Suh, 1999, 2001].

Information content generally means complexity, and it can be defined differently depending on characteristics of each design [EI-Haik, 1999; Suh, 1999]. Until now, the index that can be applied to a general design is the probability of success. El-Haik and Yang [EI-Haik, 1999] suggested measuring the design complexity with variability and vulnerability separately. Frey [Frey, 2000] suggested a calculation method of the information content of a decoupled design based on the probability of success. Kar [Kar, 2000] explained the relation between axiomatic design and the Taguchi method through information content. Shin and Park [Shin, et al. 2002] suggested a design process using the Independence and Information Axioms in axiomatic design. In a real design field, applying the Information Axiom is not easy because calculation of the information content in a multi functional requirement problem is difficult.

In this research, previous researches are well explained for calculation of information content. Also various methods for calculation of information content are proposed. They are carried out in FR or DP ranges. The methods can be used for uncoupled, decoupled and coupled designs.

2 AXIOMATIC DESIGN

A design is completed through continuous interactions between the goal set by a designer and the method for attaining the goal. Design is a form of a product or process that can satisfy the functional requirements (FRs) that a designer wants. In other words, it is an embodiment process of mapping functional requirements pertaining to a functional domain into design parameters (DPs) pertaining to a physical domain. Mapping is choosing a relevant design parameter, which satisfies a given functional requirement. The mapping process is illustrated in Fig. 1. A multitude of appropriate designs that satisfy a designer's functional requirements can be derived. The axioms offer design principles that can give the grounds for comparing a design with others or selecting one among many alternatives [Shin, et al. 2002].

According to the axiomatic principle, the essence of the design process lies in hierarchies as illustrated in Fig. 1. Designers

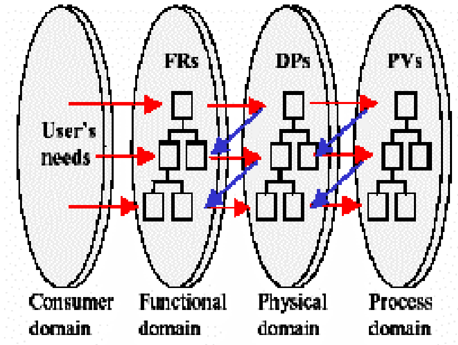


Fig. 1 Concept of domain, mapping and spaces

begin the design from comprehensive functional requirements. A design can decompose functional requirements into many hierarchies. But the decomposition of functional requirements must be carried out at the same time with the decomposition of design parameters. The zigzagging between functional requirements and design parameters is necessary because the two sets of each level are connected and mutually dependent.

Axiomatic design is a design process for a satisfactory product or design through a systematic method. A satisfactory design can be expressed as one that satisfies all the functional requirements. Therefore, a designer's role is to satisfy the requisites for a design and to define those requisites properly. In axiomatic design, a requisite for an acceptable design is to satisfy functional requirements through proper selection of design parameters. An FR is "the goal to achieve," and a DP is "the means to achieve the goal."

Axiomatic design provides a framework for choosing a good design. The two design axioms are the "tools" that are helpful for the creation of a new design. The first axiom tells us about the selection of a functional requirement. The second axiom shows a quantitative method of judging which design is more desirable. The design axioms are defined as follows:

Axiom 1: The Independence Axiom
Maintain the independence of functional requirements.
Axiom 2: The Information Axiom
Minimize the information content.

The two axioms present the most fundamental means needed to choose the best design.

For a design to be acceptable, the design must satisfy the first axiom. A design matrix is defined to pursue the relationship between FRs and DPs as follows:

$$\mathbf{FR} = \mathbf{A}\ \mathbf{DP} \tag{1}$$

where **FR** is a vector for functional requirements, **DP** is a vector for design parameters and **A** is a design matrix. If we have two FRs and DPs, Eq. (1) can be shown as follows:

$$\begin{Bmatrix} FR_1 \\ FR_2 \end{Bmatrix} = \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix} \begin{Bmatrix} DP_1 \\ DP_2 \end{Bmatrix} \tag{2a}$$

$$\begin{Bmatrix} FR_1 \\ FR_2 \end{Bmatrix} = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} DP_1 \\ DP_2 \end{Bmatrix} \tag{2b}$$

$$\begin{Bmatrix} FR_1 \\ FR_2 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} DP_1 \\ DP_2 \end{Bmatrix} \tag{2c}$$

When the Independence Axiom is satisfied, the design matrix takes the form of a diagonal matrix or a triangular matrix. A diagonal matrix in Eq. (2a) represents a perfectly uncoupled design and is the most desirable form. In this case, just one DP affects each FR because a modification on each DP only has influence on the corresponding FR. A triangular matrix in Eq. (2b) represents a decoupled design. This form of design is also a proper design, but the DPs need to be rearranged in a specific order so as to satisfy the FRs. On the other hand, an uncoupled design does not require a specific order. The third form of a design is a coupled design. This type of design is undesirable because when a DP is modified, multiple FRs are changed. There is no effective solution for undesirable change on the FRs.

The Information Axiom is related to the complexity of a design, and implies that the simpler design is the better one. In the Information Axiom, the DPs are selected according to information content. Generally, the information content is defined by the probability of success to satisfy FRs as follows:

$$I = \log_2 \frac{1}{p_s} \tag{3a}$$

$$p_s = \frac{\text{Common Range}}{\text{System Range}} \tag{3b}$$

where p_s is the probability of success. When multiple solutions are found from the Independence Axiom, the Information Axiom is utilized to find a solution with minimum information as follows:

$$I_{\min} = \min\{I_i;\quad i = 1, \cdots, n\} \tag{4}$$

where n is the number of solutions satisfying the Independence Axiom.

It is noted that the Independence Axiom should be applied first. Sometimes, it is not easy to find a single solution, which satisfies the Independence Axiom. If only one solution is found, it is the final solution. However, the general utilization of axiomatic design is illustrated in Fig. 2. First, various design solutions are investigated to satisfy the Independence Axiom. If multiple solutions are identified, the Information Axiom is utilized. The information content is the evaluation index. The one with minimum information is selected as the final solution.

3 INFORMATION CALCULATION METHOD

In case of the design Eq. (1), let design parameter DP_i have random variation δDP_i, and the functional requirement FR_i have random variation δFR_i. The range of the design parameter

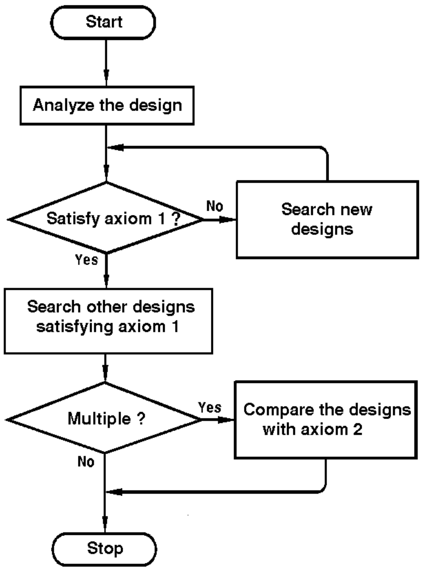


Fig. 2 Flow chart of axiomatic design

tolerance in manufacturing is $-\Delta DP_i \le \delta DP_i \le \Delta DP_i$, $i=1,\ldots,n$, $0 \le DP_i$. The success of a design means the δ**FR** variation by design parameter δ**DP** is within the tolerance range. That is $-\Delta FR_i \le \delta FR_i \le \Delta FR_i$. As shown in Eq. (3b), the probability of success p_s with two functional requirements is expressed as follows:

$$\begin{aligned} p_s &\equiv p\left(-\Delta FR_1 \le \delta FR_1 \le \Delta FR_1, -\Delta FR_2 \le \delta FR_2 \le \Delta FR_2\right) \\ &= p_1\left(-\Delta FR_1 \le \delta FR_1 \le \Delta FR_1\right) \cdot p_2\left(-\Delta FR_2 \le \delta FR_2 \le \Delta FR_2\right) \end{aligned} \tag{5a}$$

$$\begin{aligned} p_s \equiv p(&-\Delta FR_1 \le \delta FR_1 \le \Delta FR_1, \\ &-\Delta FR_2 \le \delta FR_2 \le \Delta FR_2 \mid -\Delta FR_1 \le \delta FR_1 \le \Delta FR_1) \end{aligned} \tag{5b}$$

$$p_s \equiv p\left(-\Delta FR_1 \le \delta FR_1 \le \Delta FR_1, -\Delta FR_2 \le \delta FR_2 \le \Delta FR_2\right) \tag{5c}$$

Eq. (5a) is the probability of success of an uncoupled design, Eq. (5b) is that of a decoupled design and Eq. (5c) is that of a coupled design. Eq. (5a) and Eq. (5b) are expressed with multiplication of the two probabilities and conditional probability, respectively. However, Eq. (5c) is different. Instead, the joint distribution is needed.

3.1 THE GRAPHICAL METHOD

The graphical method can be used to calculate information content. It is a calculation method of the common range from mapping the design parameter of functional requirement to the system range. In Fig. 3 the area is shown in which the system range of the design parameter is mapped into the functional range. The shape of the system range is expressed as a right-angled tetragon, a parallelogram or a diamond shape.

For example in Eq. (2b) of the decoupled design problem, let A_{ij} component of the matrix be constant and δDP_i be statistically independent. With Eq. (5b), the mapped range of the system area in the functional range by *DP* tolerance is expressed as follows:

$$-\left|A_{11}\right|\Delta DP_1 \le \delta FR_1 \le \left|A_{11}\right|\Delta DP_1 \tag{6a}$$

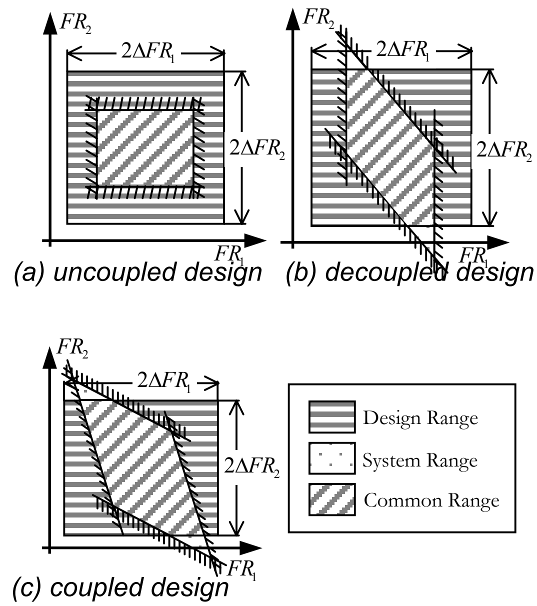


Fig. 3 Graphical representation of common range in the functional domain

$$-\left|A_{22}\right|\Delta DP_2 \le \delta FR_2 - \frac{A_{21}}{A_{11}}\delta FR_1 \le \left|A_{22}\right|\Delta DP_2 \tag{6b}$$

The graphical shape is shown in Fig. 3(b). The shape is a right angle tetragon because the design range is inside the tolerance by the decision of the designer. The area by oblique lines in Fig. 3(b) is the crossed range of the system range and the common range. The probability of success can be obtained by Eq. (3b), the information content can be calculated by Eq. (3a). The equation of probability of success by this process is obtained as shown in Eq. (7).

$$p_s = \frac{\Delta FR_1}{\left|A_{11}\delta DP_1\right|} - \frac{\left|\dfrac{A_{11}}{A_{21}}\right|\left(\left|\dfrac{A_{21}}{A_{11}}\right|\Delta FR_1 + \left|A_{22}\delta DP_2\right| - \Delta FR_2\right)^2}{4\left|A_{11}\delta DP_1\right| \cdot \left|A_{22}\delta DP_2\right|} \tag{7}$$

The information content using the graphical method is described in the present section. In the physical range, the design range is displayed by mapping of the functional requirements. The information content can be calculated with a common range by a system range of the design parameter.

3.2 THE INTEGRATION METHOD

The probability of success can be calculated by also using the integration method. Suppose that a problem has n plural design parameters. Then suppose that the distribution function of δDP_i is $f_{\delta DP_i}$, and they are independent. The probability of success is calculated as Eq. (8).

$$p_s = \int \cdots \int_{\Omega} f_{\delta DP_1} \cdots f_{\delta DP_n} \, d\delta DP_n \cdots d\delta DP_1 \qquad (8)$$

where Ω is the physical area of this equation. For example, if n=2, the integral equation in a decoupled design is calculated as Eq. (9).

$$p_s = \int_{\frac{FR_1-\Delta FR_1}{|A_{11}|}}^{\frac{FR_1+\Delta FR_1}{|A_{11}|}} \int_{\frac{FR_2-\Delta FR_2-A_{21}\delta DP_1}{|A_{22}|}}^{\frac{FR_2+\Delta FR_2-A_{21}\delta DP_1}{|A_{22}|}} f_{\delta DP_1} f_{\delta DP_2} \, d\delta DP_2 \, d\delta DP_1 \qquad (9)$$

In Eq. (9), the integral zone is the design range. In a coupled design the integration zone is made by the equations of variables for other functional requirements. Therefore, in the physical range, the method of mapping the design range of the functional requirement cannot be applied in general use. To solve this problem, we map the system range into the functional range, and perform integration in the physical range. For this conversion, coordinate transformation of the distribution function is also necessary as follows:

$$p_s = \int \cdots \int_{\Omega} f_{\delta FR_1} \cdots f_{\delta FR_n} \cdot |J| \; d\delta FR_n \cdots d\delta FR_1 \qquad (10)$$

where Ω is the design range of a functional requirement, $f_{\delta FR_i}$ is reverse function of distribution function, and J is the value of Jacobian as Eq. (11).

$$J = \begin{vmatrix} \dfrac{\partial \delta DP_1}{\partial \delta FR_1} & \cdots & \dfrac{\partial \delta DP_n}{\partial \delta FR_n} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial \delta DP_n}{\partial \delta FR_1} & \cdots & \dfrac{\partial \delta DP_n}{\partial \delta FR_n} \end{vmatrix} \qquad (11)$$

For example, the coupled design of Eq. (2c) can be converted as follows:

$$\begin{aligned} \begin{Bmatrix} \delta DP_1 \\ \delta DP_2 \end{Bmatrix} &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}^{-1} \begin{Bmatrix} \delta FR_1 \\ \delta FR_2 \end{Bmatrix} \\ &= \frac{1}{A_{11}A_{22} - A_{12}A_{21}} \begin{bmatrix} A_{22} & -A_{12} \\ -A_{21} & A_{11} \end{bmatrix} \begin{Bmatrix} \delta FR_1 \\ \delta FR_2 \end{Bmatrix} \end{aligned} \qquad (12)$$

The Jacobian can be calculated by Eq. (13) from Eq. (11)

$$\begin{aligned} J &= \begin{vmatrix} \dfrac{A_{22}}{A_{11}A_{22} - A_{12}A_{21}} & \dfrac{-A_{12}}{A_{11}A_{22} - A_{12}A_{21}} \\ \dfrac{-A_{21}}{A_{11}A_{22} - A_{12}A_{21}} & \dfrac{A_{11}}{A_{11}A_{22} - A_{12}A_{21}} \end{vmatrix} \\ &= \frac{A_{11}A_{22} + A_{12}A_{21}}{(A_{11}A_{22} - A_{12}A_{21})^2} \end{aligned} \qquad (13)$$

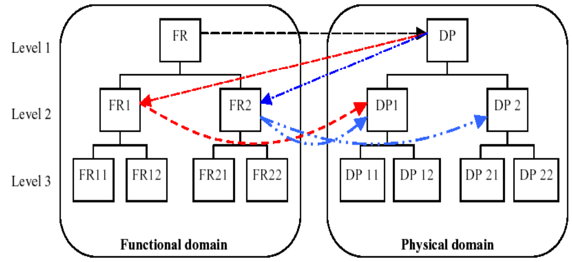


Fig. 4 Hierarchical systems of the functional requirements and design parameters

By substituting the results of (12) and (13) to (10), the general information content by the design equation can be calculated.

3.3 CALCULATION OF INFORMATION CONTENT HAVING A HIERARCHICAL STRUCTURE

In a real complex system design, the functional requirement can be decomposed into a hierarchical structure. The hierarchical structure consists of plural levels. The design equation of each level could be a decoupled design or a coupled design.

When the total design matrix is decomposed into the lowest level, the total information content can be calculated. For example, suppose we have a system having a hierarchical structure as illustrated in Fig. 4. The hierarchy has three levels and it is a decoupled design.

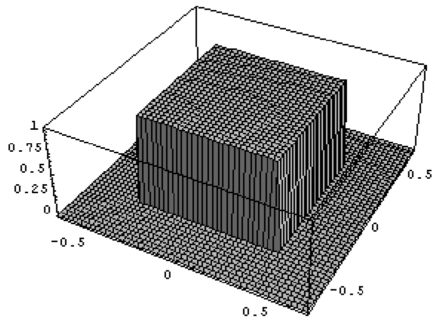
$$\begin{Bmatrix} FR_1 \\ FR_2 \end{Bmatrix} = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} DP_1 \\ DP_2 \end{Bmatrix} \qquad (14a)$$

$$\begin{Bmatrix} FR_{11} \\ FR_{12} \end{Bmatrix} = \begin{bmatrix} B_{11} & 0 \\ B_{21} & B_{22} \end{bmatrix} \begin{Bmatrix} DP_{11} \\ DP_{12} \end{Bmatrix} \qquad (14b)$$

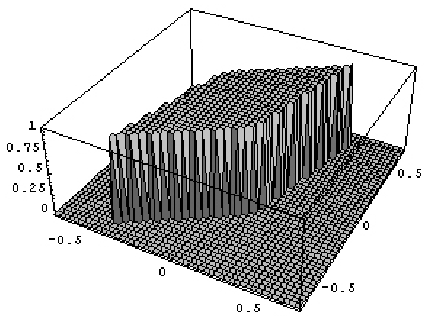
$$\begin{Bmatrix} FR_{21} \\ FR_{22} \end{Bmatrix} = \begin{bmatrix} C_{11} & 0 \\ C_{21} & C_{22} \end{bmatrix} \begin{Bmatrix} DP_{21} \\ DP_{22} \end{Bmatrix} \qquad (14c)$$

$$\begin{Bmatrix} FR_{11} \\ FR_{12} \\ FR_{21} \\ FR_{22} \end{Bmatrix} = \begin{bmatrix} B_{11} & 0 & 0 & 0 \\ B_{21} & B_{22} & 0 & 0 \\ D_{11} & D_{12} & C_{11} & 0 \\ D_{21} & D_{22} & C_{21} & C_{22} \end{bmatrix} \begin{Bmatrix} DP_{11} \\ DP_{12} \\ DP_{21} \\ DP_{22} \end{Bmatrix} \qquad (14d)$$

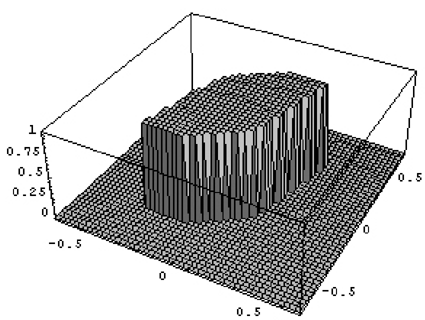
The design matrix of (14d) is called the full design matrix. The total design matrix is constructed by the decomposing process. Therefore, in the total design matrix the design factor D_{ij} can be found, which is not shown in the upper level. With the last total design matrix, the information content can be calculated by using the integration method of the previous section. In the following section, the information content is calculated for two functional requirements and two design parameters.



(a) Design Range



(b) System Range



(c) Common Range

Fig. 5 Design range, system range, common range and probability density function (pdf) of a functional requirement in the functional domain.

4 EXAMPLE OF INFORMATION CONTENT CALCULATION

In the previous section, two calculation methods for information content are explained. The information content can be calculated by using the graphical method and the integration method. In the case of a design equation that has two functional requirements and design parameters, DPs have uniformed distribution. Then the design equation is as follows:

$$\begin{Bmatrix} \delta FR_1 \\ \delta FR_2 \end{Bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix} \begin{Bmatrix} \delta DP_1 \\ \delta DP_2 \end{Bmatrix} \tag{15}$$

where the system range by tolerance, and the design range by designer is $-0.1 \le \delta DP_i \le 0.1$, $-0.4 \le \delta FR_i \le 0.4$, i=1,2. The information content can be calculated in the functional range.

4.1 CALCULATION OF INFORMATION CONTENT USING THE GRAPHICAL METHOD

In Fig. 5 the area of each domain is as follows: the system range is 0.4 and the common range is 0.35, the probability of success p_s is calculated by Eq. (3b)

$$p_s = \frac{0.35}{0.4} = 0.875 \tag{16}$$

The information content I is

$$I = \log_2\left(\frac{1}{p_s}\right) = \log_2\left(\frac{1}{0.875}\right) = 0.193(\text{bits}) \tag{17}$$

4.2 CALCULATION OF INFORMATION CONTENT USING THE INTEGRATION METHOD

Suppose that the tolerance of **DP** is $\Delta DP_1 = \Delta DP_2 = 0.1$ and the tolerance of **FR** is $\Delta FR_1 = \Delta FR_2 = 0.4$. The unit step function is used because the **DP** is uniformly distributed. Using Eq. (11), the Jacobian value is 0.1. If it is substituted into Eq. (10), and the probability of success p_s can be calculated as follows:

$$\begin{aligned} p_s = \int_{-0.4}^{0.4}\int_{-0.4}^{0.4} \frac{1}{2(0.1)} & \left[\mathrm{U}\left(\frac{4}{10}\delta FR_1 - \frac{1}{10}\delta FR_2 + 0.1\right) \right. \\ & \left. - \mathrm{U}\left(\frac{4}{10}\delta FR_1 - \frac{1}{10}\delta FR_2 - 0.1\right) \right] \\ \times \frac{1}{2(0.1)} & \left[\mathrm{U}\left(\frac{-2}{10}\delta FR_1 + \frac{3}{10}\delta FR_2 + 0.1\right) \right. \\ & \left. - \mathrm{U}\left(\frac{-2}{10}\delta FR_1 + \frac{3}{10}\delta FR_2 - 0.1\right) \right] \\ \times \frac{1}{10} & \; d\delta FR_2 \, d\delta FR_1 = 0.875 \end{aligned} \tag{18}$$

The information content is 0.193(bits). This is the same result with that of the graphical method of the previous section.

5 CONCLUSION

The calculation methods for applying Information Axiom are described. In the case where the number of functional requirement is more than two, the graphical method is not applied. The integration method can calculate the information content when the number of functional requirement is more than two. The integration method calculating most of the design cases is suggested. This method calculates by mapping the system range into the functional range. In a complex system where there is a hierarchical structure, the integration method using the total design matrix of the lowest levels is suggested. A final design can be determined by using the Information Axiom with suggested integration methods.

6 ACKNOWLEDGMENTS

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