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A UNIFYING STATISTICAL APPROACH FOR THE RECOGNITION OF INVARIANT PRODUCT SHAPES

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ABSTRACT

Every engineering system is designed for delivering its basic functions and Axiomatic design develops a list of methods able to attempt this task.
The designers through axiomatic principles have to translate Functional Requirements into Geometrical Requirements and develop verification and validation procedure able to establish if the process, controlled by Physical Variables, has produced an object satisfying the Functional Requirements.
Given that the product design cannot reasonably be assured if it is not coupled with a verification procedure that is unique, universal, repeatable, in this paper it is proposed an original approach to the geometrical control of product shape which exploits the potentiality embedded in the standard ISO/TS 17450-1 and in particular in the duality principle between specification and verification phases.
The duality principle is only one of several tools which Geometrical Product Specification and verification (GPS) project provides to the designer to achieve axiomatic quality.

Keywords: Axiomatic Design, GPS, shape recognition.

1 INTRODUCTION TO AXIOMATIC DESIGN

Every engineering system is designed for delivering its basic functions.
To satisfy the functional requirements must always be designed, according to the discussion of Pahl & Beitz [1], a clear and easily reproduced relationship between inputs and outputs; to do this it is necessary to consider at least one of the three types of conversion: material, energy and/or signal conversion (Fig. 1).

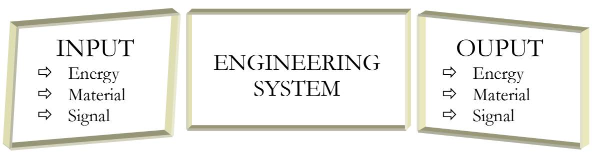

Figure 1 – The conversion of energy, material and signal.

Axiomatic Design formalizes this relationship developing a global theory to implement an optimal design.
In the following there is a brief review on principal concepts of Axiomatic Design.
Axiomatic Design recognizes four domains:

- **customer domain** where the needs of the customer are identified**;**
- **functional domain** where the object of design is stated: the customer needs are specified in terms of Functional Requirements of a product. In this domain the basic functions that the engineering system will deliver are defined ;
- **physical domain** where the physical solution is generated in term of Design Parameters that must satisfy the Functional Requirements. In this domain the conversion of energy, material and/or signal takes place**;**
- **process domain** where process variables define how the product will be produced satisfying Design Parameters.

Axiomatic design defines a mapping between every couple of adjacent domain.
Concept Design is the mapping between the customer and functional domains; **Product Design** is the mapping between functional and physical domains and finally **Process Design** corresponds to the mapping between the physical and process domains.
Two design axioms are at the basis of the Axiomatic Design and they provide a rational chance for the evaluation of the obtained solution and the choice of the best solution.

- **Axiom 1:** "states that the mapping between Functional Requirements in the functional domain and Design Parameters in the physical domain must be such that a perturbation in a particular design parameter must affect only its referent functional requirement".
- **Axiom 2:** "states that, among all the designs satisfying Axiom 1, the one with minimum information content is the best design" [2, 3].

Axiomatic design is an essential tool for design of innovative products and technologies, but its role in traditional engineering is not exhausted yet and, on the contrary, promises new interesting developments.

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2 AXIOMATIC DESIGN AND GEOMETRICAL TOLERANCING

Axiomatic Design is widely used in mechanical design and, in particular, in the geometrical definition of components, where not only the literature is very large, but also experience and standardization are consolidated.
The process linking the Functional Requirements to the Design Parameters is known and applied by mechanical designers specifically in the field of the shape control, where the respect of the design axioms is generally satisfied.
The task of the designer is to translate functional requirements into geometrical requirements. This is a complex task for Geometric Dimensioning and Tolerancing [4, 5], because it is not always possible to guarantee the one-to-one relationship between functional requirements and geometrical requirements.
Furthermore one of the most important designers task is the evaluation of the system behaviour related to the design goals. In fact the product design cannot reasonably be assured if it is not coupled with a verification procedure that is unique, universal, repeatable and therefore based in large part on mathematical criteria.
It is therefore necessary to associate to each concept in physical domain a verification and validation procedure able to establish if the process, controlled by physical variables, has produced an object satisfying Functional Requirements.
The Geometrical Product Specifications and verification (GPS) project respects the principle of Axiomatic Design and solves the problems mentioned above introducing several new concepts as: the duality principle, the extension of the uncertainty concept to the specification phase, the GPS matrix and a mathematical background to standards definition.
By means of the **duality principle** [6] the sets of operations used in the specification phase to address variability limits are in biunivocal relationship with the sets of operations used in the verification phase.

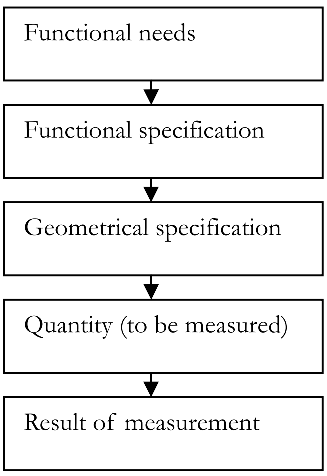

Figure 2 – Relationship between functional need and result of measurement (ISO/TS 17450-1)

Following the ISO/TS 17450-1 standard [6] the functional requirements, defined in the product design phase, are transformed in functional specifications and hence in geometrical specifications.
In the verification phase geometrical specifications address the measurement activities, by establishing an axiomatic correspondence between design and inspection phase.
The duality principle states that the measurement operations must reflect design operations in order to ensure a correct correspondence between the measured element and the nominal element.

If Axiomatic Quality (agreed upon the combination of Axiomatic Design, Robust Design and Six Sigma) recognizes two major design vulnerabilities, then GPS project recognizes two different types of uncertainty in product shape definition.
The vulnerabilities are categorized by Axiomatic Quality as

- conceptual vulnerabilities due to the violation of design principles.
- operational vulnerabilities due to the "noise factors" that are the factors not controllable by designers. This type of vulnerability is usually addressed by robust design where robustness consists on the fact that the product/process becomes insensitive to use-conditions and other uncontrollable factors.

The GPS project extends the largely accepted concept of measurement uncertainty by including several other sources which operates along the product shape definition and are generically collected as geometrical uncertainty [7]. The uncertainty management is a GPS tool which explains the discrepancies between the intended functionalities, geometrical specifications and physical measurements.
At the state of the art, the uncertainties recognized within the GPS project are:
Measurement uncertainty, which is subdivided in:

- Method uncertainty: differences existing between specification and verification operator.
- Implementation uncertainty: standard deviation of measurement process.

Geometrical uncertainty which is subdivided in:

- Correlation: incorrect or uncompleted relationship between intended functionality and geometrical control specifications.
- Specification: from incorrect or incomplete standards applied to geometrical product's definition.

Other fundamental components introduced by GPS project, which guarantee the design axioms and improve the GD&T language are:

- the **GPS matrix**, included in the ISO/TR14638 Technical Report [8], which assures the completeness in the standardization of geometrical controls. The GPS matrix consists of individual chains of standards related to specific geometrical controls along the design and verification phases in the product development process;
- the **mathematical** approach that assures unambiguity of standards as well as a successful implementation in computer based technology, thus reducing the discretionality of human interaction.

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3 SPECIFICATION AND VERIFICATION PHASES

Given that the product design cannot reasonably be assured if it is not coupled with a verification procedure that is unique, universal and repeatable, this paper illustrates an original approach to the geometrical control of product shape which exploits the potentiality embedded in the standard ISO/TS 17450-1 [6] and, specifically, in the duality principle between specification and verification phases.

3.1 CLASSIFICATION OF EUCLIDEAN SURFACES

Every mechanical object has a shape, i.e. a real surface that delimits its boundaries. The complete description of that shape – whatever this may mean – is beyond our research. In order to set up an approximate description of finite length we have to rely on measurements, thence we must face all those metrological aspects that are inherent to the data acquisition process. By contrast, nominal surfaces are completely defined in the design phase.

There is a bi-directional correspondence between nominal and real surfaces: in fact the latter is produced from the former during the manufacturing phase, while the former is associated to the latter during the verification phase. The Geometrical Dimensioning and Tolerancing (GD&T) of mechanical components and assemblies aims at specifying those aspects of the above correspondence that can affect the product functionality.

The traditional approach to GD&T essentially relies on definitions by examples. In the last decade, however, a unifying and theoretically sound new perspective emerged. Its methodological foundations were already been laid down in the seminal paper of A. Requicha [9], where the specification of tolerances was conceived as a "virtual verification procedure". More recently the International Standard Organization entrusted the technical committee ISO/TC213 [10] with the development of a complete Geometrical Product Specification (GPS) language [11]. Here a precise mathematical definition of tolerances is given in terms of variations on features of non-ideal surface models. In short, a parametric family of surfaces is defined that contains the nominal surface and, at the same time, serves as a model of the real surface. Every nominal surface feature gives rise to a specification which is formulated as a predicate on the corresponding feature of the surface model.

As stated above, the duality principle between specification and verification procedures is formulated explicitly: both do indeed consist of the same operations applied to a non-ideal surface model or to a real surface, respectively [6].

The very core of the GPS language relies upon a very elegant and powerful classification of sets of points in R^3 based on their invariance properties under the action of the twelve connected Lie subgroups of $T(3)\times SO(3)$ – the group of rigid motions [12]. Here $T(3)$ [$SO(3)$] denotes the group of translations [rotations] in R^3, respectively. Only seven such subgroups do actually leave invariant some proper subset of R^3, thus giving rise to as many classes of symmetry. Table 1 summarizes their main characteristics.

Class of Symmetry C_i (i=1,...,7)	Group of Symmetry G_i	Reference Set[a] R_i	dim(G_i)
1) Spherical	$G_1=SO(3)$	Point	3
2) Cylindrical	$G_2=T(1)\times SO(1)$	Straight Line	2
3) Planar	$G_3=T(2)\times SO(1)$	Plane	3
4) Helicoidal	$G_4=T(1)\times SO(1)$ with pitch $\mu\neq 0$	Helix	1
5) Axial	$G_5=SO(1)$	(Point, Straight Line)	1
6) Prismatic	$G_6=T(1)$	(Straight Line, Plane)	1
7) Trivial	$G_7=I_3$	(Point, Straight Line, Plane)	0

Tab. 1 – Classes of symmetry C_i

In detail, given any $S\subset\Re^3$, let Aut(S) be the group of automorphisms of S, i.e. $\mathrm{Aut}(S)=\{g\in T(3)\times SO(3): gS=S\}$; $\mathrm{Aut}_0(S)$ be the connected component of Aut(S) that contains the identity rigid motion I_3. Set S is assigned to class C_i if and only if $\mathrm{Aut}_0(S)=G_i$. In [12] it is shown that, if S is either closed or has a closed limit set, then Aut(S) is a Lie group and therefore $\mathrm{Aut}_0(S)$ is a connected Lie group. Since all such groups are listed in Table I, set S turns out to be classifiable. The above condition on S is actually so broad that it can be assumed to hold true for all practical purposes. Roughly speaking, ISO/TC213 came to a partitioning of the set of those proper subsets of R^3 that are relevant in engineering applications.

3.2 CLASSIFICATION OF MEASURED SURFACES

Let $D=\{(x_1,y_1,z_1),(x_2,y_2,z_2),...,(x_n,y_n,z_n)\}$ be a set of n measurements independently and uniformly sampled from a limited set S in presence of homoscedastic and isotropic gaussian noise. We are now in a position to look for symmetries in the underlying "true" physical object S by checking for the same symmetries on a consistent estimate $\tilde{p}$ of the (unknown) PDF $\hat{p}_S$ based on the measured points themselves.

In principle upon each symmetry group G_i (i=1,...,7) we define a semi-parametric probabilistic model M_i equipped with a set of parameters Ωi whose values define all possible specific instances of the built-in symmetry. For example, since M_6 enforces invariance with respect to translations along an arbitrary but fixed line, the orientation of that line in the lab reference system is in fact parameterized through Ω_6.

The invariance is enforced by replacing the set of measured points $D=\{(x_1,y_1,z_1),(x_2,y_2,z_2),...,(x_n,y_n,z_n)\}$ with his projection on the quotient set R^3/E_i being E_i the set of equivalent points with respect to the rigid motions $g\in Gi$.:

$$E_i=\left\{(P,Q)\in\Re^3\times\Re^3 : Q=gP \text{ for some } g\in G_i\right\}$$

Otherwise stated, the points are projected onto the quotient set R^3/E_i through a suitable parametric function $V_i(\cdot;\Omega_i)$.

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In practice however only models M_1, M_5 and M_7 are defined exactly as just advised. All seven competing models are described at length in [12].
The models assembled this way are quite heterogeneous and of largely different complexities. Any attempt to rank them according to their likelihood:

$$L\left(M_i\right) = \max_{\Omega_i} \tilde{p}\left(D \mid M_i, \Omega_i, V_i\left(D; \Omega_i\right)\right)$$

would tend to penalize the simplest ones. Some kind of regularization is therefore in order. Several solutions to this problem are reported in literature that can be applied to our methodology as well. Here we chose to stay stick to the frequentist approach and pursue a very basic leave-one-out strategy just for the sake of clarity. Models are thus ranked according to the following criterion:

$$L'\left(M_i\right) = \prod_{j=1}^{n} \tilde{p}\left(x_j, y_j, z_j \mid M_i, \tilde{\Omega}_i^{(j)}, V_i\left(D_j; \tilde{\Omega}_i^{(j)}\right)\right)$$

where $D_j = D \setminus \{(x_j, y_j, z_j)\}$ $\forall j \in \{1,\ldots,n\}$ and $\tilde{\Omega}_i^{(j)}$ is the maximum likelihood estimate of Ω_i based on D_j, namely:

$$\tilde{\Omega}_i^{(j)} = \arg\max_{\Omega_i} \tilde{p}\left(D_j \mid M_i, \Omega_i, V_i\left(D_j; \Omega_i\right)\right).$$

3.3 SEGMENTATION OF COMPLEX SURFACES

It is possible to extend the proposed classification of invariance sets in R^3 by providing a methodology for the decomposition of complex objects in their elementary surfaces.
In fact it can be easily proved that the collection of two or more sets of points, each of them belonging to any of the invariance classes, can be reclassified in one of the seven classes of invariance. [13,15]

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
C_1	C_5 C_1						
C_2	C_7 C_5	C_7 C_6 C_2					
C_3	C_5	C_7 C_5 C_6	C_6 C_3				
C_4	C_7	C_7 C_4	C_7	C_7 C_4			
C_5	C_7 C_5	C_7 C_5	C_7 C_5	C_7	C_7 C_5		
C_6	C_7	C_7 C_6	C_7 C_6	C_7	C_7	C_7 C_6	
C_7	C_7	C_7	C_7	C_7	C_7	C_7	C_7

Table 2 – Composition of two sets of points belonging to the seven invariance classes

This reclassification can be used to create complex products from simpler parts. Table 2, used in geometrical tolerancing, shows the possible results deriving from the composition of two set of points belonging to the seven invariance classes.
The same table can be used to decompose complex products in simpler parts, in fact Table 2 shows that each invariance class admits a finite number of decompositions, thus suggesting a recursive procedure for the partitioning of a complex shape in its elementary constituents.
From a procedural viewpoint the set of points is assigned to the proper invariance class, and then tests on the same set of point researches the decompositions admissible on the invariance model.
Each set of points derived from the decomposition is further decomposed to provide simpler objects. The procedure ends when no more decomposition is possible. The probabilistic approach easily implements this procedure because it focuses on the measured data without requiring any a-priori hypothesis.
The application of the probabilistic approach to the set of measured points should provide the collection of the simplest invariant elements, which constitute the product shape, without requiring any other information than the data set. Obviously a complete scanning of a set of measured points can be very time consuming even on fast CPU. However, the efficiency of the probabilistic analysis increases if some additional information is provided, for example, the candidate list of invariance classes to be tested in the decomposition procedure.

4 THE EXPERIMENTAL TESTS

We tested our method on seven simple surfaces demonstrating the ability of algorithms in classifying each surface by identifying its symmetries.[14]
In this paper we tested our method on a complex surface composed by a cylindrical and a portion of a sphere whose center is on the cylinder axis.
From a theoretical point of view the algorithm could recognize at the first step (first ranking on the complex surface) an axial symmetry C_5. Following the decomposition in Table 2 an axial symmetry can be further view as the composition of eight different couple of invariant class.
The theory based on Topologic
In particular we are interested in the decomposition of the axial surface in two set of points, one belonging to the cylindrical symmetry and the other one belonging to the spherical or axial symmetry.
The surfaces analyzed are showed with their different sizes in Table3.

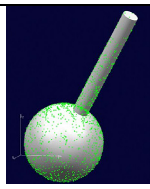	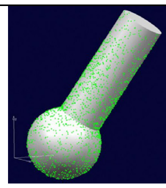	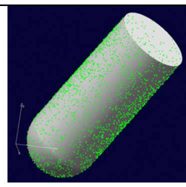
S1)	S2)	S3)
Sphere radius:50 mm	Sphere radius:50 mm	Sphere radius:50 mm
Cylinder height. 200 mm	Cylinder height 200 mm	Cylinder height 200 mm
Cylinder radius:10 mm	Cylinder radius:25 mm	Cylinder radius:50

Table 3 – Surfaces analyzed in the experimental test

We analyzed sets of point of dimension n=750 sampled uniformly from the three complex shapes mentioned above.

In the first case (case S1) the maximum log-likelihood value is achieved for the spherical class C_1 because of the large difference between the number of points sampled from the two features.

r1 =

1.0000	-12.3967
5.0000	-12.9073
7.0000	-13.8391
2.0000	-13.8735
6.0000	-14.1779
3.0000	-14.3093
4.0000	-14.3351

The recognized sphere is characterized by a radius 50 mm as showed in Figure 3. The waviness of the Probability Density Function for radius values ranging from 50 mm to 200 mm reveals the existence of the points sampled along the thin cylinder connected to the sphere. This small set of sampled points is not correctly interpreted by the ranking procedure.

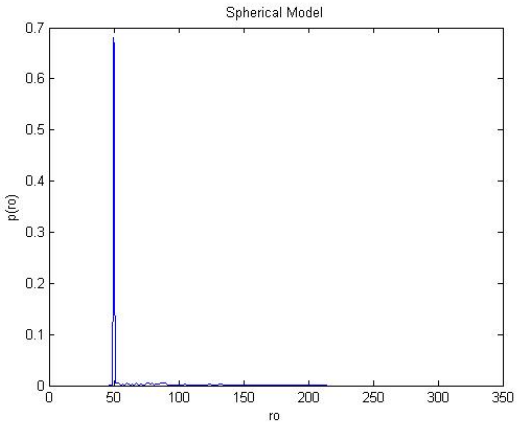

Figure 3 – Sphere radius PDF.

In case S2 a first ranking between the seven invariant classes provides an axial invariance:

r1 =

5.0000	-12.8045
2.0000	-13.7804
7.0000	-14.2084
4.0000	-14.4042
6.0000	-14.4304
3.0000	-14.5746
1.0000	-15.3786

The axial shape recognized presents a joint distribution of his axis and radius as showed in Figure 4. The three-dimensional PDF is not immediately intelligible but the shape of the sampled set is clearly knowledgeable in Figure 5, which is a two-dimensional representation of the same PDF illustrated in Figure 4.

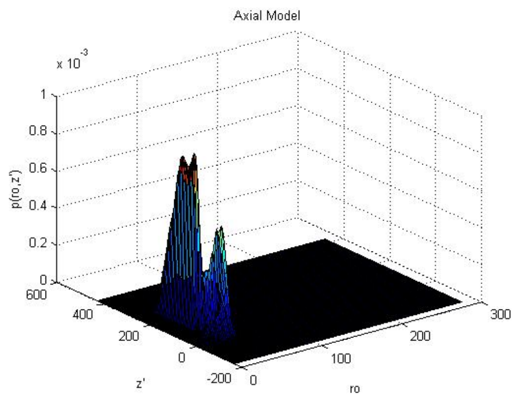

Figure 4 – Joint PDF of axis and radius for the recognized axial symmetry (case S2).

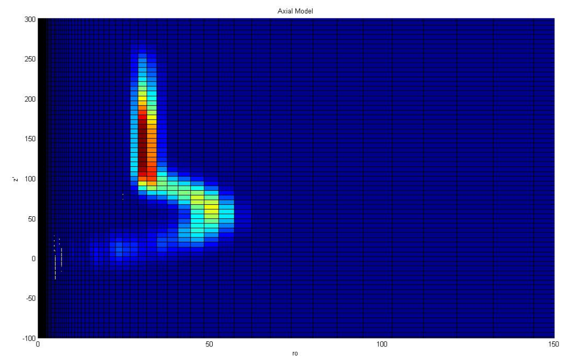

Figure 5 – Two-dimensional view of the joint PDF of the recognized axial symmetry (case S2).

In a second ranking among all possible segmentation of axial symmetry: {[C_1;C_1], [C_1;C_2], [C_1;C_5], [C_2;C_2], [C_2;C_5], C_5} the maximum log-likelihood value is achieved for the decomposition in spherical and cylindrical features:

r2 =

2.0000	-10.6380
3.0000	-11.6055
5.0000	-11.6645
4.0000	-12.0431
6.0000	-12.8098
1.0000	-13.2686

The probabilistic description of the recognized Cylinder geometrical characteristics are presented in Figure 6. Figure 6a represents the probabilistic description of the cylinder length, while Figure 6b represents the probabilistic description of the cylinder radius.

The probabilistic description of the recognized Spherical radius are presented in Figure 7.

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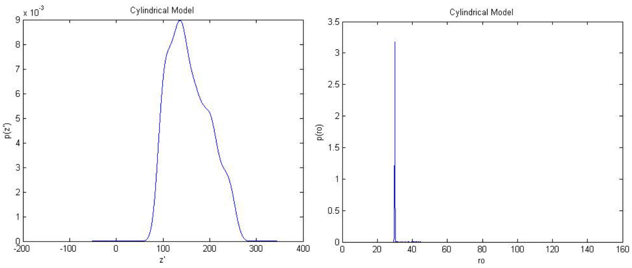


a) b)

Figure 6 – PDF's of axis (a) and radius (b) for the cylindrical symmetry (case S2).

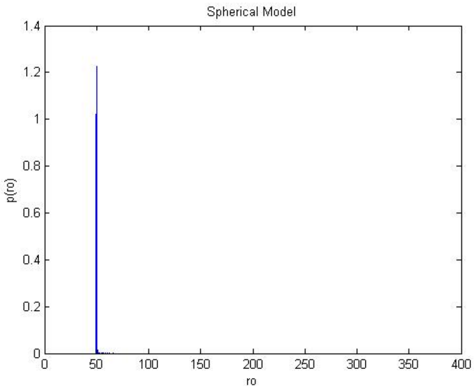


Figure 7 – PDF's of radius for the spherical symmetry (case S2).

As last case study we consider the **case S3)** where again an axial symmetry is recognized in the first ranking procedure:

r1 =

5.0000	-13.3523
2.0000	-13.5979
4.0000	-14.4296
6.0000	-14.4447
7.0000	-14.6205
3.0000	-14.8728
1.0000	-15.4492

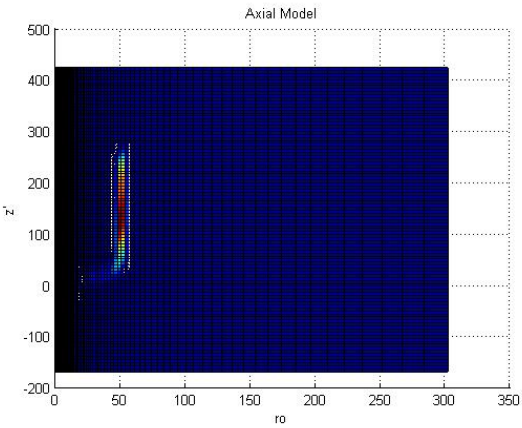


Figure 8 – Joint PDF of axis and radius for the recognized axial symmetry (case S3).

The axial shape recognized presents a joint distribution of his axis and radius as showed in figure 8.
In the case S3, the second ranking (between all possible segmentation of axial symmetry) recognizes again a decomposition of axial symmetry in spherical and cylindrical features:

r2 =

2.0000	-12.4475
5.0000	-12.4973
4.0000	-12.7215
3.0000	-13.2284
6.0000	-13.3523
1.0000	-15.4894

The sphere recognized is characterized by a radius 50 mm as showed in Figure 9 and Cylinder shape by geometrical characteristics presented in Figure 10a and Figure 10b respectively for cylinder length and cylinder radius.

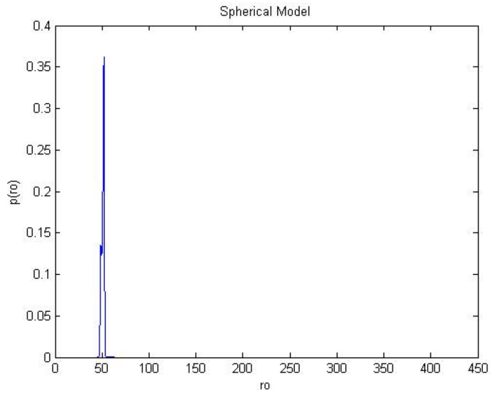
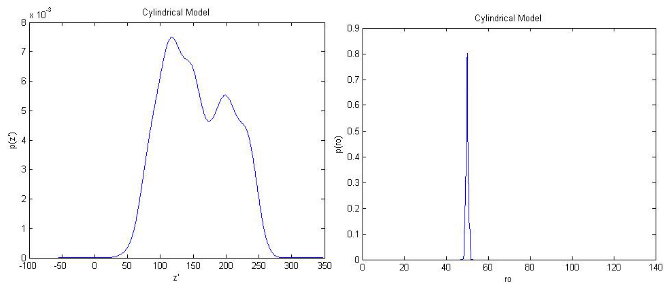


Figure 9 – PDF's of radius for the spherical symmetry (case S3).



a) b)

Figure 10 – PDF's of axis (a) and radius (b) for the cylindrical symmetry (case S3).

5 CONCLUSIONS

In this paper a probabilistic approach is proposed as a tool to support the future development of axiomatic quality in product shape definition. Specifically, the proposed approach addresses a verification and validation procedure which is able to control if

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the Functional Requirements defined in functional domain are satisfied by the manufactured object.
This method takes in account all new principles introduced by ISO/TC213 in the GPS project and in particular it relies on the duality principle, described in the ISO/TS17450 standard as a biunivocal correspondence between the design and verification activities performed in the product development process.
The proposed methodology adopts a statistical approach because it provides a unified description of the shape of a product along its life-cycle.
We tested the proposed methodology not only on the seven invariance classes [14] with the goal of identifying the surface class, but also on a particular complex surface composed of a cylinder and a portion of a sphere whose center is on the cylinder axis. Such test demonstrates that the proposed approach could be applied to the decomposition of complex surfaces in its elementary features, which is a problem not presently solved by the ISO/TC213 experts but representing a fundamental step within the framework of Geometrical Product Specifications and Verification.

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