Proceedings of ICAD2006
4th International Conference on Axiomatic Design
Firenze – June 13-16, 2006
ICAD-2006-16

FUSION OF BELIEF IN AXIOMATIC DESIGN

Michele Pappalardo
mpappalardo@unisa.it
Department of Mechanical Engineering, University of Salerno, Italy
Via Ponte Don Melillo, 84084, Fisciano (SA) Italy
Tel +39-089-964245

ABSTRACT

Axiomatic Design methodology is developed using matrix methods to analyze the innovation needs into design parameters. The method is based on two design axioms from which it is possible to derive rules for improve design activities. The first axiom, called the *Independence Axiom* states the preservation of the independence of functional requirements. The second axiom, called the *Information Axiom* states the highest probability of success. The information is related to the Bayesian probability of satisfying of a given set of constraint. When there is little information on which to evaluate a probability or when that information is non-specific, ambiguous, or in conflict the Bayesian model cannot be used. Then can be used the Dempster-Shafer's reasoning system. In this formalism probability values are assigned to sets of possibilities rather than single events. From sets can be obtained the measure of the information of the Axiomatic Design methodology.

Keywords: Design axiom, Information, Belief

1 INTRODUCTION

Axiomatic design methodology forms the conceptual design process into a continuous and measurable innovation, improving quality of design. The design process gives the structure necessary for the transformation of the qualitative needs, often stated in non engineering terms, to the real products.
Designers typically follow the needs of customers understanding their requirements creating and selecting solutions. Using the axiomatic design process, designers can use all their existing design tools and arrive at a successful new design. This transformation is achieved through the application of scientific knowledge to the problems. Using previous design databases the design process generate several alternatives to be evaluated frequently.
Usually the design process is subdivided into a series of phases with specific, evaluations to made between these phases. Each evaluation determines whether the phase needs to be repeated, or if the designer needs to go back on one or more phase.
Nam P. Suh (1990) proposes an axiomatic method for highly complex designs. The design process optimize elements using a set $\{\boldsymbol{FR_i}\}$ of functional requirements and a set $\{\boldsymbol{DP_j}\}$ of physical parameter. He proposed two axioms that could help to have a good design.

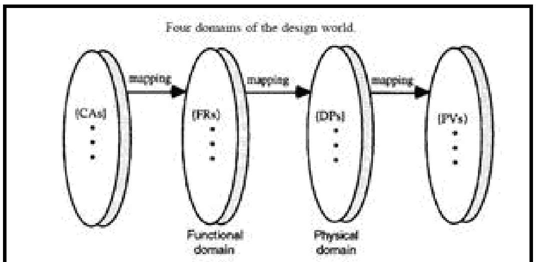


Figure 1 Nam P. Suh

The relation of $\{\boldsymbol{FR_i}\}$ with the $\{\boldsymbol{DP_i}\}$ is mathematically expressed as $\boldsymbol{FR_i} = \boldsymbol{f}(\boldsymbol{DP_i})$. The design process is reduced to a series of mappings from the design's functional requirements into the design's parameter space. The mapping process between the domains is repeated several times, with the results that previous design parameters determine the next set of functional requirements.
The domains are defined by the two vectors:

$$\{\boldsymbol{FR}\} = \boldsymbol{Vector\ of\ \ functional\ \ domain}$$
$$\{\boldsymbol{DP}\} = \boldsymbol{Vector\ of\ \ physical\ domain}$$

The relation between these two domain in matrix notation is written as

$$\{FR\} = [A]\{DP\} \qquad (1)$$

were $[A]$ is the design matrix.

In the problems in which the $\{FR_i\}$ depend from non linear functions, the equation (1) can be written in differential form

$$\{dFR\} = [A]\{dDP\}$$

The elements of design matrix can be written as $A_{ij} = \partial FR_i / \partial DP_j$ and the design matrix is

$$[A] = \begin{bmatrix} \partial FR_1/\partial DP_1 & .. & .. & \partial FR_1/\partial DP_j \\ .. & .. & .. & .. \\ \partial FR_i/\partial DP_1 & .. & .. & \partial FR_i/\partial DP_j \end{bmatrix}$$

A small change in any parameter may cause a deviation in the functional requirement

$$\Delta FR_i = \frac{\partial FR_i}{\partial DP_j}\Delta DP_j$$

In linear design $A_{ij} = \partial FR_i / \partial DP_j$ are constants. We have

$$FR_i = \sum_{i=1}^{n} A_{ij} DP_j$$

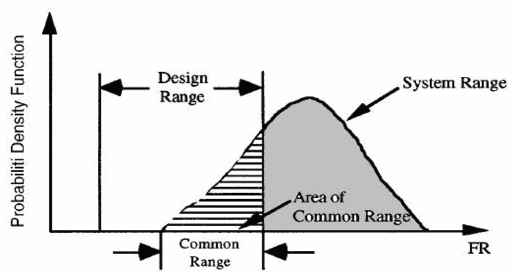
In (1), the diagonal matrix is a special cases. The design matrix $[A]$, in general, is a rectangular array of values.

In the design process Nam P. Suh has indicated two axioms, on functional requirement, in order to examine the actions of planning. The axioms are:

Axiom 1: The independence Axiom. Maintain the independence of functional requirement.

Axiom 2: The information Axiom. Minimize the information contents of the design.

The first axiom states that the independence of functional set $\{FR_i\}$ must be always maintained. The information axiom states that the best design has the minimum information and the minimum of functional requirements. For to compare two design, one can compare the information content of the two design which can satisfy the functionally parameters. The information content can be described by means similar to the Wiener-Shannon's theory.



Ranges in a plot of the pdf of functional requirements

Figure 2 Nam P. Suh

The elements of a design matrix $[A]$ can be constants or functions, with the consequence that the design may be non linear. Mathematical techniques can transform a matrix, but the physical significance of the elements A_{ij} can be lost. An ideal design matrix is a square diagonal matrix with each *FR* related one to one to a single *DP*. The uncoupled tolerance for a DP_i is $\Delta DP_i = \Delta FR_i / A_{ii}$. The propagation of tolerance for a decoupled design with a lower triangular matrix $n \times n$, is expressed as

$$\Delta DP_i^* = \frac{\Delta FR_i - \sum_{j=1}^{n} \left| A_{ij} \Delta DP_j \right|}{A_{ii}} \qquad (2)$$

From equation (2) it is evident that is $\Delta DP_i \geq \Delta DP_i^*$. The consequence is that decoupled design has less tolerance than an uncoupled design, and the increase of the order of design matrix makes the last DP_i's tolerance smaller. If the number of DP_s is greater than the number of FR_s , then the design is redundant.

When the number of DP_s is less then the number of FR_s , then the coupled design cannot be satisfied. When number of $DP_s < FR_s$, using E.T. Jaynes' MaxEnt Principle, it is possible to obtain approximate solutions consistent with boundary conditions.

In ideal design each functional requirement must be linked to one design parameter, and vice versa each design parameter can satisfy one (or more) functional parameter. From the system of equations it turns out obvious that with the number of $DP_s < FR_s$ it is possible to have only approximate solutions. In this situation the number of DP_s is insufficient to achieve all the FR_s in exact mode. A good design have the minimum information content. Analyzing the information of a design it is possible to understand the physical influence of constraints. The information axiom states that the best design has then minimum information and the minimum of functional requirements.

2 LEARNING USING RULES OF DEMPSTER-SHAFER

Appling the *Information Axiom* states the highest probability of success. The information is related to the Bayesian probability. When there is little information on which to evaluate a probability or when that information is non-specific, ambiguous, or in conflict the Bayesian model cannot be used.

A method for handling data in the presence of uncertainty with qualitative values is the theory of Dempster-Shafer. The theory of Dempster-Shafer (DS) is a method for reasoning under uncertainty. The Ds model include the Bayesan probability as special case, and introduce the *belief function* as lower probabilities and the *plausibility function* as upper probabilities. The Numerical measure in presence of uncertainty may be assigned to set of propositions as well as to a single proposition. The probabilities are apportioned to subsets and the mass v_i can move over each element. Let the finite non empty set $\mathbf{\Theta} = \{\mathbf{x_1}, ..\mathbf{x_n}\}$ be the *frame of discernment* which is the set of all the hypothesis. The basic probability is assigned in the range $[0,1]$ to the 2^n subset of Θ consisting of a singleton or conjunction of singleton of n elements x_i. The basic probability is a function which assign the weight to the subset so that $\sum_{A \subseteq \Theta} m(A) = 1$ and $\mathbf{m(\Phi) = 0}$.

The lower probability $\mathbf{P_*(A_j)}$ is defined as $P_*(A_j) = \sum_{A_{ji} A_i} m(A_j)$. And the upper probability $\mathbf{P^*(A_j)}$ is defined as $P^*(A_j) = 1 - \sum_{A_{ji} A_i} m(A_j)$.

The $\mathbf{m(A_i)}$ values are the independent basic values of probability inferred on each subset $\mathbf{A_i}$. The belief function of set M it is given by

$$Bel(M) = \sum_{A_i iM} m(A_i) \qquad Pl(M) = \sum_{A_i \cap M \neq \Theta} m(A_i)$$

The *evidential interval* that provides a lower and upper bound is

$$EI = [Bl(M), Pl(M)] \qquad (2)$$

If m_1 and m_2 are the independent basic probabilities from the independent evidence, and $\{A_{1i}\}$ and $\{A_{2j}\}$ the sets of focal points, then the theorem of Shafer gives the rule of combination. Let m_1 and m_2 two independent basic probabilities from the independent evidence. If

$$\sum_{A_{1i} \cap A_{2j} \neq \Phi} m_1(A_{1i}) \; m_2(A_{2j}) > 0$$

then

$$m(A) = (m_1 \oplus m_2)(A) \overset{def}{=} \frac{\sum_{A_{1i} \cap A_{2j} = A_k} m_1(A_{1i}) m_2(A_{2j})}{1 - \sum_{A_{1i} \cap A_{2j} = \Phi} m_1(A_{1i}) m_2(A_{2j})}, \qquad 3)$$

$$A \neq \Phi, (m_1 \oplus m_2) = 2^{\Theta} \rightarrow [0,1]$$

gives the rule for combining two or more probability are given from independent evidence.

3 APPLICATION OF THE INFORMATION AXION IN SELECTION OF A SYNCRO-MESH

The change gear in an automotive transmission is a device that provides different gear drive ratio between the engine and driving wheels of the vehicle, the principal function being to enable the vehicle to accelerate from rest to a wide range of speed while the engine operates within its most affective range. For a correct change of gear drive ratio, on meshing the gears, the toothed wheels must have its teethe whit the same velocity. The correct mesh is obtained using a mechanical device that synchronises the gears using a syncro ring, with a conic surface, that drags to match the same speed to the teethe of the couple of wheels. Designers uses various kinds of devices to synchronise cars. A very simple gear change with device to synchronise is shown fig. 1. With the push of a lever, the axial shift of mechanical elements of the synchroniser is possible,. The force that presses on the lever is opposed, from the inside springs of the device.

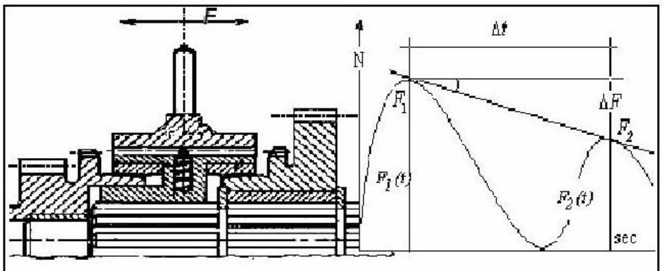


Figure 3

In order to change a gear ratio the driver pushes on the lever and, <u>with a first variable force</u> $F_1(t)$ he gets the piece with conic surface, with friction on the surface of the opposite conic bodies, that the teeth of the opposite gear are at the same speed. After which <u>with a second variable force</u> $F_2(t)$ he gets the connection of opposite gears and allows the automatic change of gear ratios. The two force are applied in a little interval of time Δt. The quality of the device for change of gear ratio depend on max values of F_1, F_2 and Δt the distance of time of max forces. A fundamental parameter is

$$K = \frac{F_1 - F_2}{\Delta t} \quad (\text{N/sec}) \; ; \quad \Delta F = F_1 - F_2 \qquad 4)$$

with $[0,1]$ and F_2 values max of viable forces on the lever.

Drivers have many different wishes which depend on age, education, employment, etc. The classification of features of change gear give three large sets:

Synchroniser with Δt very narrow, $\Delta F = F_1 - F_2$ narrow and F_1 medium: Drivers notice, by lever, only one pulse during the shift of different gear drive ratio: The relation between forces is: $F_2 \leq 0.5 F_1$.

Synchroniser with Δt medium, $\Delta F = F_1 - F_2$ narrow and F_1 medium: Drivers notice, by lever, two pulse during the shift of different gear drive ratio.

Synchroniser with Δt narrow, $\Delta F = F_1 - F_2$ large and F_1 large: Drivers notice, by lever, one pulse during the shift of different gear drive ratio.

The aim goal of the designers is that of giving the drivers an automobile transmission that is easy to shift from different gear drive ratios. Every driver classifies the change gear in a subjective way. The main characteristics of a gearbox are:

Typology x_1: Synchromesh, usually with one syncro ring, with which the drivers notice, by lever, only one pulse during the shift of different gear drive ratio, which is more often used in small cars.

Typology x_2: Synchromesh, usually with three syncro ring, with which the drivers notice, by lever, two pulse during the shift of different gear drive ratios, which is generally in medium cars .

Typology x_3: Synchromesh, usually with two syncro ring, with which the drivers don't notice any pulse by lever during the shift of different gear drive ratio, which is more often with two syncro ring and used in car with large engine and with sporting features.

According to the analysis of industrial manufacture of a large number of cars the situation is constituted by a mixed typology. The set A of result is: $\{x_1\}$, $\{x_2\}$, $\{x_3\}$ and $\{x_2 x_3\}$ with the distribution
$m_1(\{x_1\}) = 0.1$, $m_1(\{x_2\}) = 0.25$, $m_1(\{x_3\}) = 0.05$ $m_1(\{x_1, x_2\}) = 0.6$.
In order to design a new passenger car, two sets of drivers, consulted on their wishes of features on change gear, gave two sets of results.
First set B of result, in DS theory, is : $m_2(\{x_1, x_2\}) = 0.2$ $m_2(x_2) = 0.5$, $m_2(\{x_2, x_3\}) = 0.3$. Second set C of result, in DS theory, is: $m_3(\{x_2\}) = 0.4$, $m_3(\{x_3\}) = 0.2$, $m_3(\{x_2, x_3\}) = 0.25$, $m_3(X) = 0.15$
The results are contradictory because there is in drivers have many of different wishes depending on age, education, employment, etc.
Numerical measure of probability is assigned to set of propositions. In order to minimize the information content we must have data in terms of probability on single typology of solutions. The equation of information is

$$I = \sum_{i=1}^{n} \log_2 p_i \qquad 5)$$

where p_i is the probability of DP_i (number of syncro ring) satisfying FR_i (pulse during the shift). Since there are Bayesian solutions the total information content is the sum of all individual measures.
The problem can be analyzed using the DS theory . For combination of degrees of evidence from the sources using the DS theorem.
We calculate the belief of each set in Table 1.

	EXP 1		EXP. 2	COMBINED EV. $m_{1,2} = m_1 \oplus m_2$			EXP. 3	COMBINED EV. $(m_1 \oplus m_2) \oplus m_3 = m_{1,2,3}$		
Focal elements	$m_1(x_i)$		$m_2(x_i)$	$m_{1,2}(x_i)$	Bl(.)	Pl(.)	$m_3(x_i)$	$m_{1,2,3}(x_i)$	Bl(.)	Pl(.)
$\{x_1\}$	0.10	0.10	0.00	0.026	0.026	0.02	0.00	0.0050	0.0050	0.0050
$\{x_2\}$	0.25	0.25	0.50	0.720	0.72	0.96	0.40	0.8101	0.8101	0.9240
$\{x_3\}$	0.05	0.05	0.00	0.020	0.02	0.26	0.20	0.0713	0.0713	0.1852
$\{x_1 \cup x_2\}$	0	0.35	0.20	0.000	0.74	0.98	0.00	0.0000	0.8106	0.8151
$\{x_1 \cup x_3\}$	0	0.15	0.00	0.000	0.04	0.04	0.00	0.0000	0.0760	0.8902
$\{x_2 \cup x_3\}$	0.60	0.90	0.30	0.240	0.98	0.98	0.25	0.1139	0.9943	0.9943
$\{x_1 \cup x_2 \cup x_3\}$	0.	1	0.00	0.00	1	1	0.15	0.00	1	1

Table 1

Step 1 : The rule of the combination of the evidence of Shafer of gives: $m_1 \oplus m_2(\{x_1\}) = 0.026$, $m_1 \oplus m_2(\{x_2\}) = 0.720$, $m_1 \oplus m_2(\{x_3\}) = 0.020$ and $m_1 \oplus m_2(\{x_2 \cup x_3\}) = 0.240$
The Belief of x_2 is 0.72. The Disbelief of x_2 is the Belief of $\{x_1 \cup x_3\}$ is 0.04. The Plausibility of x_2 is (1-0.04) =0.96. The evidential interval is $EI_{Step1}(x_2) =$ $[Bel(x_2), Pl(x_2)] =$ $[0.72, 0.96] = 0.24$. As more and more evidence accumulate in the DS theory, the interval of a set tends to narrows.
Step 2: The combination of evidence gives: $(m_1 \oplus m_2) \oplus m_3(\{x_1\}) = 0.0050$, $(m_1 \oplus m_2) \oplus m_3(\{x_1\}) = 0.8101$, $(m_1 \oplus m_2) \oplus m_3(\{x_1\}) = 0.0713$, $(m_1 \oplus m_2) \oplus m_3(\{x_2 \cup x_3\}) = 0.1139$.
The Belief of the x_2 is 0.8101. The Disbelief x_2 is the Belief of $\{x_1 \cup x_3\}$ = 0.0760. The Plausibility of x_2 is (1-0.0760) =0.924.The new evidential interval is $EI_{Step2}(x_2) = [Bel(x_2), Pl(x_2)] = [0.8101, 0.9240] = 0.1139$ From the accumulation of evidence, the evidential interval is narrow with $EI_{Step2} < EI_{Step2}(x_2)$ (0.1139<0.2400).
From the data of experiment 1 we have that the probability is conjunction of singleton of 2 elements x_i : $(m_1 \oplus m_2) \oplus m_3(\{x_2 \cup x_3\}) = 0.0.1139$.

	x_i	x_2	x_3	$x_2 \cup x_3$
DS Solution	0.0050	0,8101	0.0713	0.1139
Laplace-Bayes	0.0050	0,8670	0,1273	

Table 2

In applying Axiom 2 we needs a Bayesian distribution of probability. The principle of Laplace suggest us the a fair division of the probability on the two elements x_i: x_2, x_3.

	EXP 1	Bayesian	EXP. 2	Bayesian	$m_1 \oplus m_2 = m_{12}(x_i)$	Bayesian	EXP 3	Bayesian	$(m_1 \oplus m_2) \oplus m_3 = m_{123}(x_i)$	Bayesian
Focal elements	$m_1(x_i)$		$m_2(x_i)$				$m_3(x_i)$			
$\{x_1\}$	0.10	0.10	0.	0.1	0.02	0.02	0.00	0.05	0.0050	0.0050
$\{x_2\}$	0.25	0.55	0.50	0.60	0.72	0.86	0.40	0.575	0.8101	0.8607
$\{x_3\}$	0.05	0.35	0.	0.30	0.02	0.12	0.20	0,375	0.0713	0,1283
$\{x_1 \cup x_2\}$	0		0.20		0.		0.		0.	
$\{x_1 \cup x_3\}$	0		0.		0.		0.		0.	
$\{x_2 \cup x_3\}$	0.60		0.30		0.24		0.25		0.1139	
$\{x_1 \cup x_2 \cup x_3\}$	0.		0.		1		0.15			

Table 3

From *DS* analysis the final probability of x_2 is *(p=0.8670)* which on the basis of the *Second Axiom* minimize the information content (Table 4). The typology x_2 is the most relevant, which results that drivers prefer synchromesh on which notice, by lever, two near pulse during the shift of different gear drive ratio.

$$I = \sum_{i=1}^{n} \log_2 x_i$$

$$Ix_1 = -(\mathrm{LOG}(0.1) + \mathrm{LOG}(0.1) + \mathrm{LOG}(0.02) + \mathrm{LOG}(0.05) + \mathrm{LOG}(0.005)) = 16.81124283$$

$$Ix_2 = -(\mathrm{LOG}(0.55) + \mathrm{LOG}(0.6) + \mathrm{LOG}(0.86) + \mathrm{LOG}(0.8101) + \mathrm{LOG}(0.8607)) = 1.620092363$$

$$Ix_3 = -(\mathrm{LOG}(0.35) + \mathrm{LOG}(0.3) + \mathrm{LOG}(0.12) + \mathrm{LOG}(0.375) + \mathrm{LOG}(0.1283)) = 7.408271725$$

4 CONCLUSIONS

With low information on which to evaluate a probability or when that information is non-specific, ambiguous, we have used the Dempster-Shafer's reasoning system. In this formalism we have assigned probability values to sets of possibilities. From sets we have obtained the measure of the information useful for selecting a synchromesh using the Second Axiom of the Axiomatic Design methodology.

5 REFERENCES

[1] Siddal, N. *Probabilistic Engineer Design,* M.Dekker JHG N.York 1983.

[2] Hubka, V. and Eder, W.E. *Theory of Technical Systems* 1988 Springer-Verlag, New York.

[3] Pahl, G. Beitz, W. *Engineerig Design.* 1988 The design Council , London.

[4] Nam P. Suh: *The Principles of Design.* Oxford University Press New York 1990.

[5] Smets, Ph. *What is Dempster-Shafer's Model-* Advances In The Dempster-Shafer Theory Of Evudence- Yager,Fedrizzi Kacprzyk- Editors, John Wiley e Sons USA- Professional Computing-1994

[6] N. Kapur, K. Kesavan, *Entropy Optimization Principles*, Acad. Press Inc. Boston 1992.

[7] Pugh, S. *Creative Innovative Products Using Total Design.* 1996 Addison-Wesley Publishing Company, Inc. New York.

[8] Pappalardo, M. *Information in Design Process,* 2nd International Conference on Planned Maintenance, Reliability and Quality - Oxford , England 2nd -3rd April 1998.

[9] Jaynes E.T. *Probability Theory: The logic of Science.* http://omega/.albany. edu: 8008/ Jaynes.

[10] Nam P. Suh *Axiomatic Design-.* MIT-Pappalardo Series in Mechanical Engineering Oxford University Press New York 2001.

[11] Pappalardo, M. - *Measure of independence in soft design-* Journal of Material Processing Technology – 124 (2002)-ELSEVIER.

[12] Pappalardo M., *Axiomatic approach in the analysis of data for describing complex shapes*, proceedings of ICAD 2004 International Conference on Axiomatic Design, 21-24 of June 2004, Seoul – South Korea.

[13] Basem Said *Axiomatic Quality*, 2005 Wiley