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AXIOMATIC-DESIGN APPROACH TO THE MAIN-BEARING CONFIGURATION OF A JET ENGINE WITH SEVERAL SHAFTS

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ABSTRACT

The best configuration for the main bearings of a multi-shaft jet engine is studied by means of the Axiomatic Design. This paper shows how Axiomatic Design reduces the number of initial possible solutions from several hundreds to one. It also shows how the implementation of the Information Axiom as an initial customer's need affects the formulation of the functional requirements and constraints, and hence the generation of a final solution. Information Axiom is used for screening the best solution among those prescribed by the Independence Axiom. Current solutions for three-shaft and two-shaft engines are discussed and compared with the best solution, which in this context is the one given by the axiomatic approach. This paper also discusses how constraints may lead to slightly different solutions.

Keywords: independence axiom, information axiom, reliability, jet engine, turbofan, main bearings.

1 INTRODUCTION

Axiomatic Design [Suh, 1990 and Suh, 2001] is a methodology that increases the value of products and it has proven to be applicable to diverse design problems with great success. A review of solved cases can be found in [Kulak *et al.*, 2010].

In the context of Axiomatic Design, design is defined as the generation of maps between domains. There are four domains: the customer, functional, physical and process domains. The mapping between the customer and functional domains leads to the definition of two lists: the list of functional requirements and the list of constraints. The mapping between the functional domain and the physical domain is characterized by the design matrix. In order to obtain the best design, Axiomatic Design establishes which rules must be satisfied by both mappings. A first rule imposes that i) functional requirements are a minimum set of independent requirements that completely characterize the functional needs, and ii) constraints are bounds on acceptable solutions. A second rule imposes that the best mapping must satisfy two axioms:

Independence Axiom: Maintain the independence of the functional requirements.

Information Axiom: Minimize the information content of the design.

The Information Axiom selects, from those solutions that satisfy the Independence Axiom, the solution that has the greatest probability of success. In addition to the Axioms, constraints play a fundamental role in Axiomatic Design because they allow the independence of the functional requirements. When a requirement cannot be added to the list of functional requirements (for example, because it breaks the independence of the rest of the requirements), it must be considered as a constraint. Therefore, the mapping between the customer and functional domains is not unique. For this reason, the selection of the functional requirements and constraints (in the functional domain) that best characterize the customer's motivation (in the customer domain) is crucial for obtaining the best solution (in the physical domain). Correct selection of functional requirements and constraints is one of the crucial tasks as it has been referenced in the literature [Suh, 1990; Suh, 2001 and Brown, 2005].

In this paper Axiomatic Design is used to establish the best configuration for the main bearings of jet engines with several shafts. This is the customer's motivation (in the customer domain). The number, type and location of such bearings define the solution (in the physical domain). One of the objectives of this paper is to study how the definition of functional requirements and constraints (in the functional domain) affects the screening made by the Axioms (in the physical domain). For this purpose, the paper is divided into three main sections. Section 2 describes the solution given by Axiomatic Design for a standard mapping from the customer to the functional domain. This standard mapping does not include reliability as a functional requirement because reliability is taken into account by the Information Axiom (maximize the probability of success). On the other hand, Section 3 modifies this mapping by using another list of functional requirements and constraints that includes reliability as an explicit functional requirement. It is shown in the paper that the methodology used to impose the accomplishment of the Independence Axiom gives two different solutions; the first one accomplishes Corollary 3 (integration of physical parts) [Suh, 1990], whereas the second one does not. Finally, Section 4 discusses the assumptions that led to the differences obtained in both approaches and gives a solution for the jet-engine problem.

2 MAIN-BEARING CONFIGURATION OF A JET ENGINE WITH SEVERAL SHAFTS

This section illustrates how Axiomatic Design can be used to select the main roller bearings of a jet engine with *N* concentric shafts. It shows how both axioms lead to a frozen design giving the number, type, and relative position of the roller bearings that support the shafts inside the engine. Fig. 1 shows a scheme with a feasible solution for the problem. Each shaft is supported by two ball roller bearings that are connected to the adjacent inner shaft, while the central shaft is connected to the case. Obviously, special features of the engine (such as length, rigidity, interferences, etc.) will separate the adopted solution for a given engine from the one fixed by the Axiomatic Design. However, the solution addressed by the axiomatic approach can be considered as the target one.

This section is divided into the three domains proposed by the Axiomatic Design: the customer domain, where the motivation and the needs of the customer are described; the functional domain, where the list of functional requirements and constraints that define the customer's motivation are fixed; and the physical domain, where the set of proposed solutions are described in terms of the design parameters. At the end of the section, the axioms are applied in order to select a solution.

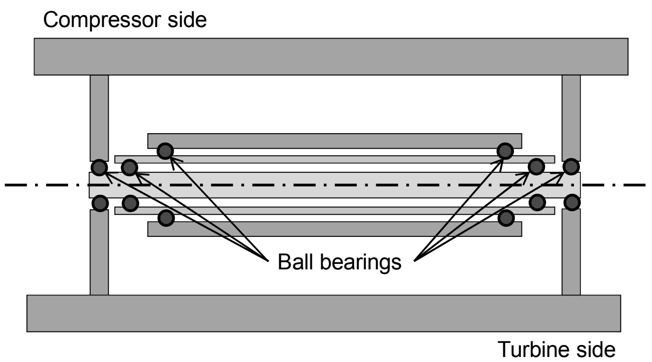

Figure 1. A possible configuration for a three-shaft engine that has two identical ball bearings per shaft. Each shaft is joined to the inner one, except for shaft 1, which is attached to the case.

2.1 CUSTOMER DOMAIN

A statement that describes the customer's motivation is: "establish the best configuration for the main roller bearings that support the shafts of a turbofan". Note that in this statement the number of concentric shafts is not fixed. However, it is useful to think about an engine like the one schematized in Fig. 1, which is a three-shaft engine where one shaft connects the low pressure turbine with the low pressure compressor (the fan), and the other two shafts connect the high and medium pressure turbines with the high and medium pressure compressors. Obviously, with the engine in operation, the aerodynamic loads on the turbomachinery produce axial forces over each one of the shafts. On the other hand, the aircraft imposes a set of forces over the rear and front mounts of the engine. Fig. 2 shows a scheme with the main forces acting on the shafts and the case of the engine. In this figure the loads imposed by the main bearings are not drawn since determining them is the object of this design problem. After this quick argumentation, the customer's motivation can be reformulated as the following customer's need: "Maintain the relative position of the shafts and the case allowing the axial rotation of the shafts".

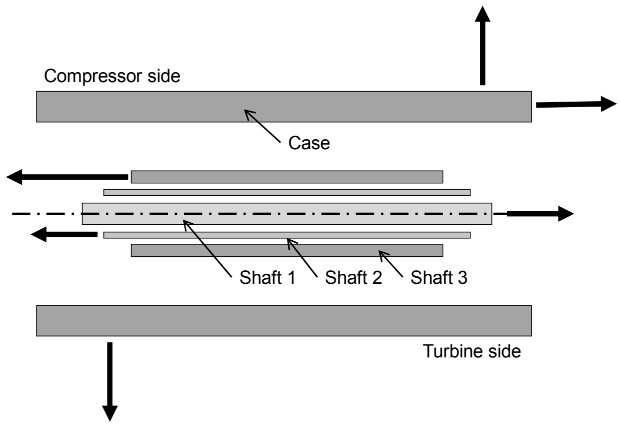

Figure 2. Main forces acting on the shafts and the case of the engine (bearings are not considered).

2.2 FUNCTIONAL DOMAIN

Each shaft is a solid part, therefore the rigid body theory advises us that each shaft has six degrees of freedom. This fact allows us to rewrite the customer's need for each shaft as the following:

List of customer's needs (list 1): Avoid any axial translation of the shaft, avoid any radial translation of the shaft, avoid any radial rotation of the shaft, and allow any axial rotation of the shaft.

List 1 is a list of needs but it is not a list of functional requirements. Following the definition of functional requirements given in the introduction, functional requirements must be a minimum set of independent needs. List 1 is not independent because any radial rotation over any arbitrary point of the shaft would induce a radial translation on other points of the shaft. There are two ways to fix this problem. Lists 2 and 3 describe them.

List of customer's needs (list 2): Avoid the axial translation, avoid the radial translation of a point A of the shaft, avoid any radial rotation around the point A, and allow any axial rotation of the shaft.

List of customer's needs (list 3): Avoid the axial translation, avoid the radial translation of a point A of the shaft, avoid the radial translation of another point B of the shaft, and allow the axial rotation of the shaft.

List 2 remains dependent because the radial translation and the radial rotation depend on each other through the point chosen as A. List 3 fixes this problem because, for small displacements, the radial translations of points A and B are independent. (One point cannot be displaced in the radial direction without moving the other.) However, elements in list 3 are not independent. Since the shaft has a finite diameter, the points where the shaft must be supported are on the surface not in the axis. Thus, points A and B are on the surface, and hence the axial rotation of the shaft is coupled with the radial displacement of both points. (For example, an axial rotation of 90 degrees changes the radial position of points A and B.) This means that the need “allow the axial rotation of the shaft” is coupled with both “avoid the radial translation of point A” and “avoid the radial translation of point B”. Therefore, (following the definition of functional requirements given in the introduction) the need “allow the axial rotation” is considered a constraint. Finally, the definition of the design problem in the functional domain has the following functional requirements and constraints (see Table 1):

List of functional requirements: Avoid the axial translation of the shaft, avoid the radial translation of a point of the shaft (point A), and avoid the radial translation of another point of the shaft (point B).

List of constraints: Allow the axial rotation of the shaft.

These lists of functional requirements and constraints can be replicated for each one of the shafts without breaking the conditions of being minimum and independent. Note that functional requirements and constraints have been obtained for a rigid body (infinite stiffness is assumed). A real shaft has a finite stiffness that could require more than two points of control to accomplish the customer’s motivation. The introduction of a third point (point C) in the list would break the independence of the list. Axiomatic Design recommends increasing the stiffness of the shaft (mainly to flexion) until the third point of control could be removed from the list. In this way “provide enough stiffness to flexion” could appear as a new constraint in the list of constraints.

List of constraints: Allow the axial rotation of the shaft, and provide enough stiffness to flexion.

Table 1. Functional requirements for one shaft.

FRs	Description
FR_1	Avoid the axial translation
FR_2	Avoid the radial translation of point A
FR_3	Avoid the radial translation of point B

2.3 PHYSICAL DOMAIN

The solution adopted in the physical domain is a set of roller bearings. A roller bearing is able to avoid radial and axial displacements and allows axial rotation. Note that, in general, a roller bearing is able to support radial and axial forces as well as small radial torques in the point where it is mounted. This means that each roller bearing introduces three design parameters (see Table 2): axial force, radial force, and radial torque. The design matrix for a set of *n* roller bearings appears in Fig. 3. These bearings can be used to connect the shaft to another shaft and/or to the case.

Table 2. Design parameters for a roller bearing.

DPs	Description
DP_1	Axial force
DP_2	Radial force
DP_3	Radial torque

		Bearing 1			Bearing 2			Bearing 3			…
		DP_1	DP_2	DP_3	DP_1	DP_2	DP_3	DP_1	DP_2	DP_3	…
Shaft 1	FR_1	X			X			X			…
	FR_2		X	x		X	x		X	x	…
	FR_3		X	x		X	x		X	x	…

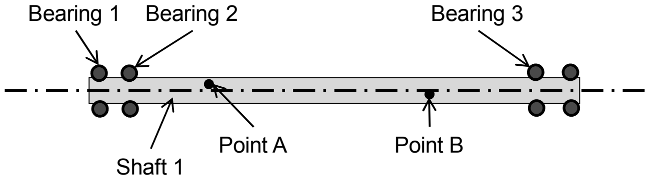

Figure 3. Design matrix and scheme for a set of roller bearings supporting one shaft.

Fig. 3 shows the design matrix for an arbitrary position of the bearings (i.e., their positions do not have to coincide with points A and B). This means that all of the radial forces of the bearings affect FR_2 and FR_3. Because the thickness of a roller bearing is much less than the length of the shaft, the radial torque that a roller bearing can support is small. Therefore, the design matrix has dominant effects (the elements marked with a capital X), elements whose order of magnitude is much less than the dominant ones (the elements marked with a lowercase x), and elements whose effect is negligible (elements with a blank).

The number of bearings over the shaft, their type, and their positions must be defined by the designer. Following Axiomatic Design, the design matrix given in Fig. 3 does not represent a good design because it has more design parameters than functional requirements and because the design matrix is coupled.

2.4 INDEPENDENCE AXIOM

Axiomatic Design shows that the previous design can be improved if the design matrix becomes diagonal. To achieve this objective, we will retain one dominant element in each row: the one that allows the matrix to be rearranged to be diagonal. Fig. 4 shows the result.

		Bearing 1			Bearing 2			Bearing 3			…
		DP_1	DP_2	DP_3	DP_1	DP_2	DP_3	DP_1	DP_2	DP_3	…
Shaft 1	FR_1	X									…
	FR_2		X								…
	FR_3					X					…

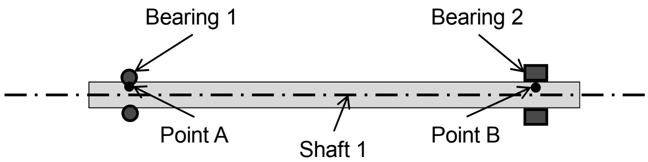

Figure 4. Matrix and scheme of a decoupled design for one shaft.

The decoupled design matrix obtained from the Independence Axiom states that: 1) only two bearings must be used, 2) the radial force introduced by one bearing must affect only the radial displacement of one point, and 3) only one bearing must support axial forces. Condition 1) states that the set of bearings must be a pair.. To achieve condition 2), bearing 1 must be placed over point A, and bearing 2, over point B. To achieve condition 3), one roller bearing must be a ball roller bearing, and the other, a cylinder roller bearing. Thus, the solution imposed by Axiom 1 is feasible. This solution uses a couple of bearings (one is a ball bearing, and the other a cylinder bearing), so the design parameters can be rewritten as shown in Table 3.

Table 3. Design parameters for a couple of bearings.

DPs	Description
DP_1	Axial force of the ball bearing
DP_2	Radial force of the ball bearing
DP_3	Radial force of the cylinder bearing

For an arbitrary number of shafts, say *N*, the solution above indicates that each shaft requires a couple of bearings. However, nothing is stated about the relative position of the bearings between the shafts and the case.

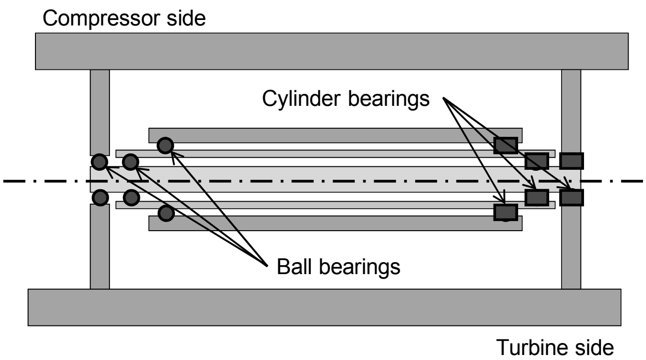

Figure 5. Arrangement whose design matrix is the one given by Fig. 6.

		Couple 1			Couple 2			Couple 3			…
		DP_1	DP_2	DP_3	DP_1	DP_2	DP_3	DP_1	DP_2	DP_3	…
Shaft 1	FR_1	**X**			**X**						…
	FR_2		**X**			**X**					…
	FR_3			**X**			**X**				…
Shaft 2	FR_1				**X**			**X**			…
	FR_2					**X**			**X**		…
	FR_3						**X**			**X**	…
Shaft 3	FR_1							**X**			…
	FR_2								**X**		…
	FR_3									**X**	…
…	…	…	…	…	…	…	…	…	…	…	…

Figure 6. Design matrix for *N* shafts and a set of *N* couples of bearings.

A feasible solution is the configuration shown in Fig. 5. The design matrix of this configuration is the one given in Fig. 6. Again, Axiom 1 states that this solution is not a good one because its design matrix is not diagonal. In order to obtain a better design we retain in each row the dominant effect that changes the matrix into a diagonal one. Fig. 7 shows the result. A scheme of the physical arrangement for this configuration is drawn in Fig. 8. Thus, Axiom 1 has promoted a solution where all the shafts are connected to the case.

		Couple 1			Couple 2			Couple 3			…
		DP_1	DP_2	DP_3	DP_1	DP_2	DP_3	DP_1	DP_2	DP_3	…
Shaft 1	FR_1	**X**									…
	FR_2		**X**								…
	FR_3			**X**							…
Shaft 2	FR_1				**X**						…
	FR_2					**X**					…
	FR_3						**X**				…
Shaft 3	FR_1							**X**			…
	FR_2								**X**		…
	FR_3									**X**	…
…	…	…	…	…	…	…	…	…	…	…	…

Figure 7. Decoupled design matrix for N shafts and a set of N couples of bearings.

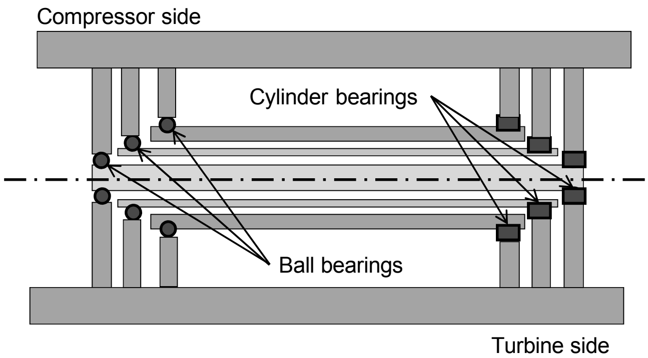

Figure 8. Arrangement for the decoupled design matrix given in Fig. 7.

2.5 INFORMATION AXIOM

In the solution fixed by the Independence Axiom (Fig. 7), there is still an ambiguity. Since the two bearings are not symmetrical (one bearing is a ball bearing and the other is a cylinder bearing), the position of the couple with respect to the inlet and outlet sections of the engine must be specified. The configuration in Fig. 8 is one possibility, but the configuration with the roller bearings in the side of the compressor is equally valid. To fix this question we will use the Information Axiom. The Information Axiom states that the probability of success must be maximized. As a consequence, reliability must be introduced in the formulation of the problem.

The demands on ball bearings are greater than the demands on cylinder ones because: 1) the former supports a greater load (they must support axial and radial loads, whereas cylinder ones only support radial loads), and 2) the stress concentration is greater in a ball bearing than in a cylinder one. (The contact for a ball is a point, whereas the contact for a cylinder is a line.) Thus, for a given level of loads, and a given lifetime, the reliability of ball bearings is less than the reliability of cylinder bearings. Besides, the greater the temperature, the greater the losses in mechanical properties (for example, hardness decays if temperature increases). Thus the compressor side is a less dangerous environment than the turbine (or combustor) side. In order to increase the probability of success, the weaker elements should be positioned in the less aggressive environment. So, ball bearing must be near compressors, and cylinder bearings must be near turbines. Therefore, the Information Axiom chooses the configuration in Fig. 8 as the best one.

2.6 ASSESSMENT OF THE SOLUTION

As shown, Axiomatic Design has frozen a conceptual design that satisfies the customer's motivation. The frozen configuration is the nearest to the ideal design that the system allows. Obviously, although Axiomatic Design marks this solution as optimum, industry can exploit others. For example the TRENT family of turbofans developed and exploited by Rolls-Royce, which are turbofans with three shafts, exhibits a configuration very similar to the one obtained in this paper but not identical [Rolls-Royce, 2010]. Note that this family of engines covers a huge range of sizes: it is used for the propulsion of the aircrafts Airbus A330-200, and -300 (Trent 700), Boeing 777-200, -200ER, and -300 (Trent 800), Airbus A340-500, and -600 (Trent 500), Airbus A380-800, and -800F (Trent 900), Boeing 787 (Trent 1000), and Airbus A350-800, and -900 (Trent 1700). The aircrafts Airbus A340-200, and -300 also use the engine CFM56-5C, which is an engine with two shafts [CFM, 2010]. Obviously, it is not the purpose of this paper to discuss the particularities of all these engines. In general, the main differences appear in the shaft that supports the fan, because it normally has three main bearings, and a roller bearing connected to another shaft. Therefore, a particularity of design adopted by the industry is the location of roller bearings between shafts. Since this solution is not optimum from the point of view of Axiomatic Design, Axiomatic Design advises to revise the constraints that are leading to such design.

A non-optimum design for a three-shaft engine, like the one in Fig. 5, has six conflictive elements in the design matrix. As can be seen in Fig. 6, removing one shaft reduces the number of off-diagonal elements from 6 to 3. Besides, if there is only one bearing between the shafts, and if this is the cylinder one, the number of off-diagonal elements is again reduced from 3 to 1. This configuration is almost ideal (there is only one problematic element in the design matrix). A detailed study of a similar two-shaft configuration with a roller bearing between shafts appears for a small turbojet engine in [Hsia-Wei *et al.*, 2004].

For the optimal solution obtained previously (see Fig. 8), it is also interesting to note that both forces, the axial force and one of the radial forces, are supported by the same bearing (the ball bearing). Thus this bearing is satisfying two functional requirements at the same time. Hence, the solution previously obtained also satisfies Corollary 3 (Integration of physical parts [Suh, 1990]). This corollary states that design features must be integrated in a single physical part if FRs can be independently satisfied in the proposed solution. As the next section shows, solutions that accomplish this corollary have a lower reliability than those that do not. However, the next section shows that, in a high-speed shaft, centrifugal forces over the balls (or the cylinders) in the bearing avoid the separation of radial and axial forces. Hence, the solution previously obtained is the best one for a jet engine.

3 MAXIMUM RELIABILITY AS A CUSTOMER'S NEED

3.1 REFORMULATION OF THE DESIGN PROBLEM

The Information Axiom establishes that the probability of success must be maximized. Therefore, the probability of success of each physical part must be as high as possible. Hence, the customer's motivation might have been written as: "establish the configuration for the main bearings of a jet engine that supports the shafts and that assures that the probability of success is maximum".

3.2 CUSTOMER DOMAIN

In the aeronautical environment, reliability is of the highest interest. This means that probability of success (or reliability) is always a need that describes the customer's motivation. Thus for this customer, Axiom 2 is a reminder of this need. In this case, the customer's motivation is: "Maintain the relative position of the shafts and the case allowing any axial rotation of the shafts with maximum reliability".

3.3 FUNCTIONAL DOMAIN

The new motivation can be explicitly formulated if a new need is added to list 3. The result is the following list.

List of customer's needs (list 4): Avoid the axial translation, avoid the radial translation of a point A on the shaft, avoid the radial translation of another point B on the shaft, allow the axial rotation, and obtain the highest reliability.

List 4 has been obtained from the list 3 discussed above by adding the new need associated with reliability. This new need is not independent of the others because the reliability decreases as long as the other needs are stricter. Although all of the needs are required to define the motivation of the customer, the Information Axiom and the customer's need dictate that reliability is the most important one. Hence, the motivation could be expressed in the functional domain as:

List of functional requirements: obtain the highest reliability.

List of constraints: Avoid the axial translation of the shaft, avoid the radial translation of a point on the shaft (point A), avoid the radial translation of another point on the shaft (point B), and allow the axial rotation.

Because the reliability is obtained only when a solution is provided in the physical domain, the list of functional requirements can be rewritten as:

List of functional requirements: obtain the highest reliability for each part in the physical domain.

Table 4. Functional requirements.

FRs	Description
FR_1	Maximum reliability of part 1 in the physical domain
FR_2	Maximum reliability of part 2 in the physical domain
FR_3	Maximum reliability of part 3 in the physical domain
...	...

3.4 PHYSICAL DOMAIN

As in the previous section, the solution adopted in the physical domain for fulfilling the functional requirements and constraints is a set of roller bearings. Again, each roller bearing introduces three design parameters: axial force, radial force, and radial torque. However, now the functional requirement is the reliability of each bearing, so there are more design parameters that affect this functional requirement than before. For example, bearing temperature and rotational speed also affect reliability. Let us call operational parameters to this set of variables affecting the reliability. They are collected for one bearing in Table 5. The resulting design matrix for a set of shafts connected to each other by a set of ball bearings is represented in Fig. 9. In this design matrix, the intensities of the relationships between operational parameters and functional requirements have been labelled with a number: 1, 2 or 3. (The larger the number, the stronger the dependency.) Therefore, following the Independence Axiom, it is better to remove from the design matrix the higher numbers in first place. This has been done for two different solutions, (see Figs. 10 and 11, which are, respectively, the design matrices of the designs given in Figs. 8 and 12)).

Table 5. Operational parameters for a roller bearing.

Ps	Description
P_1	Axial force
P_2	Radial force
P_3	Radial torque
P_4	Others such as temperature and rotational speed

			Operational Parameter																										
			S_1													S_2													
			B_1				B_2				B_3				…	B_1				B_2				B_3				…	
			P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4	…	P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4	…	
Reliability	S_1	B_1	1	1	2	3	1	1	2		1	1	2		…	1	1	2		1	1	2		1	1	2			
		B_2	1	1	2		1	1	2	3	1	1	2		…	1	1	2		1	1	2		1	1	2			
		B_3	1	1	2		1	1	2		1	1	2	3	…	1	1	2		1	1	2		1	1	2			
		...	...	...	...		...	...	...		...	...	...			...	...	...		...	...	...		...	...	...			
	S_2	B_1													…	1	1	2	3	1	1	2		1	1	2			
		B_2													…	1	1	2		1	1	2	3	1	1	2			
		B_3													…	1	1	2		1	1	2		1	1	2	3		
																...	...	...		...	...	...		...	...	...			

Figure 9. Design matrix for *N* shafts and a set of *n* roller bearings. All bearings are ball bearings and each shaft is connected to the immediately inner shaft, except the shaft 1, which is connected directly to the case.

			Operational Parameter																										
			S_1													S_2													
			B_1				B_2				B_3				…	B_1				B_2				B_3				…	
			P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4	…	P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4	…	
Reliability	S_1	B_1	1	1		3									…														
		B_2						1		3					…														
		B_3													…														
		...	...	...	...		...	...	...		...	...	...																
	S_2	B_1													…	1	1		3										
		B_2													…						1		3						
		B_3													…														
		...														...	...	...		...	...	...		...	...	...			

Figure 10. Design matrix for *N* shafts and a couple of bearings per shaft. One bearing is a ball bearing and the other one is a cylinder bearing. Each shaft is connected only to the case.

			Operational Parameter																										
			S_1													S_2													
			B_1				B_2				B_3				…	B_1				B_2				B_3				…	
			P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4	…	P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4	…	
Reliability	S_1	B_1	1			3									…														
		B_2						1		3					…														
		B_3											1	3	…														
		...	...	...	...		...	...	...		...	...	...																
	S_2	B_1													…	1			3										
		B_2													…						1		3						
		B_3													…											1	3		
		...														...	...	...		...	...	...		...	...	...			

Figure 11. Design matrix for *N* shafts and three bearings per shaft. Each shaft has a bearing supporting pure axial loads, and two bearings supporting pure radial loads. All bearings are cylinder bearings and each shaft is connected only to the case.

3.5 INDEPENDENCE AND INFORMATION AXIOMS

As a first approach, the reliability of a roller bearing follows a Weibull distribution [Harris, 2001 and Benavides, 2010]:

$$\ln \frac{1}{R} = \ln \frac{10}{9} L^e \left(\frac{F}{C} \right)^{pe} \quad (1)$$

where R is the reliability, L is the lifetime, and F is the force.

Parameters C, p and e are model parameters which depend respectively on the detailed design (and manufacturing), type of contact, and material. We will assume that the probability of success can be approached in a first attempt as the reliability (i.e., the probability of success during a lifetime L supporting a force F is R). As a consequence the information content can be expressed in NATs as:

$$I = -\ln R = \ln \frac{10}{9} L^e \left(\frac{F}{C} \right)^{pe} \quad (2)$$

The information contents for bearings that support 1) only axial force, 2) only radial force, and 3) axial plus radial forces, are respectively given by Eqs. (3), (4), and (5).

$$I_a = \ln \frac{10}{9} L^e \left(\frac{F_a}{C} \right)^{pe} \quad (3)$$

$$I_r = \ln \frac{10}{9} L^e \left(\frac{F_r}{C} \right)^{pe} \quad (4)$$

$$I_{ar} = \ln \frac{10}{9} L^e \left(\frac{\sqrt{F_a^2 + F_r^2}}{C} \right)^{pe} \quad (5)$$

Therefore the information content of a design given by the design matrix of Fig. 10 is I_{ar}, while the information content of the design given by the design matrix of Fig. 11 is I_a+I_r. In order to compare the information content of both designs we will use the dimensionless difference of information content given by:

$$\frac{I_a + I_r - I_{ar}}{I_a} = 1 + \left(\frac{F_r}{F_a} \right)^{pe} - \left(1 + \frac{F_r^2}{F_a^2} \right)^{\frac{pe}{2}} \quad (6)$$

This expression can be rewritten as the following function:

$$f(x) = 1 + x^q - (1+x)^q = x^q - qx + 0(x^2) \quad (7)$$

$$\frac{df(x)}{dx} = qx^{q-1} \left[1 - (1 + x^{-1})^{q-1} \right] \quad (8)$$

Because the first derivative is always negative if $q>1$ (or always positive if $q<1$), it is possible to prove that $f(x)$ is always negative when $q>1$ (respectively, positive when $q<1$). A ball bearing has a typical value for e near 1.12 and for p near 3, so that $q=pe/2$ is near 1.68, which is greater than one. This means that the information content of the design in Fig. 10 is always greater than the information content of the design in Fig. 11, i.e., $I_a+I_r-I_{ar}<0$. Thus, the Information Axiom leads us to replace the ball bearing in each shaft by a set of two cylinder bearings (one for supporting axial loads and the other for supporting radial loads). This solution is only valid for low speed shafts, because Eq. (3) does not include radial forces due to centrifugal loads over the roller elements. This is the solution with maximum reliability that accomplishes both axioms. (Note that it does not accomplish Corollary 3, and that it is only valid for low rotational speeds.)

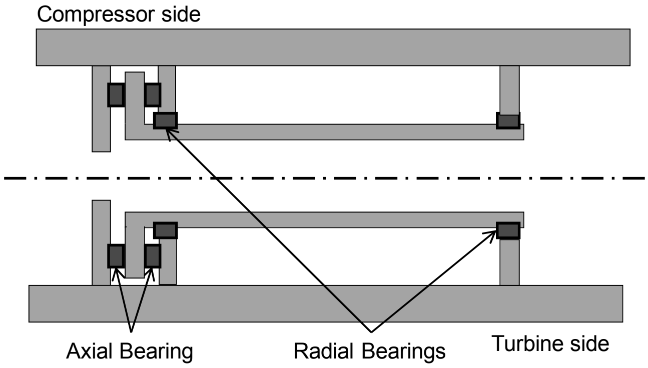

Figure 12. Scheme of one shaft supported by a set of three cylinder bearings. This solution comes from the design matrix in Fig. 11.

4 DISCUSSION

The previous sections have shown that both axioms produce different solutions depending on the set of functional requirements and constraints. The main difference between both approaches is the incorporation of the reliability as an explicit requirement. Although having two solutions could seem a contradiction, it is not. The reason is that the Axiomatic Design is a guide for the creativity process. This guide has leaded us to two different solutions that satisfy the Independence Axiom. (Note that the solution in Fig. 12 also satisfies the formulation of the design problem given in Section 2.) On the other hand, Information Axiom tends to select the design in Fig. 12 as the best one. However, this option is only correct for slow machines. For high speed shafts (such as jet engine shafts) it is very difficult to include a roller bearing without radial forces because the centrifugal load over each roller element is always present. This means that it is impossible to separate the axial and the radial loads and hence the solution in Figs. 11 and 12 is not feasible. Therefore, the best configuration is the one represented in Figs. 7, 8, and 10, which is also the one chosen by the industry.

This technological application of the Axiomatic Design illustrates the power of this methodology as a tool to save time and cost in the design process. In effect, the number of initial configurations grows as a power of 2. For a configuration with N shafts and n roller bearings in each shaft, there are the followings physical restrictions: 1) at least two bearings of the configuration must be touching the case, and 2) at least one bearing in each shaft must be a ball bearing. This means that 1) there are N ball bearings that can be placed at the compressor side or at the turbine side, 2) there are $N(n-1)$ bearings that can be ball bearings or cylinder bearings, and 3) there are $(N-1)n$ bearings that can be touching the case or touching the inner shaft. Therefore, the number of initial configurations is $2^{N(2^n-1)-n+1}$. With 3 shafts and 2 bearings in each shaft, there are 256 initial configurations! Axiomatic Design selects only one of them for being considered in the detailed design.

5 CONCLUSION

This paper shows that Axiomatic Design is applicable for obtaining the best configuration of the main bearings of a jet engine with several shafts. The application of this methodology to a three-shaft engine has reduced the number of initial cases from 256 to 1. Therefore, it proves that Axiomatic Design is a design technique of high added value because, with a minimum consumption of resources, it reduces the number of initial possibilities from a huge number to one. Besides, in the absence of other constraints or needs, this solution establishes a target for industrial developments.

This paper shows how the list of functional requirements and constraints can be easily modified if the Information Axiom is included as an initial need. In this case, the result is a configuration with three bearings per shaft. This solution accomplishes the Independence Axiom. However, depending on the rotational speed of the shafts, the solution obtained from the new set of functional requirements and constraints may or may not comply with the Information Axiom. For low speed shafts, the information content of the new solution is lower than the previous one (with two bearings per shaft). For a case with higher rotational speeds, which is the case of jet engines, the new solution is not physically acceptable. Therefore, the target solution is a set of two bearings per shaft where one bearing is a ball bearing placed at the compressor side, the other is a cylinder bearing placed at the turbine side, and where both bearings are fixed to the case.

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