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METHODOLOGICAL CONTRADICTIONS SOLVED BY THE LINEARITY THEOREM: THE VACUUM CLEANER AS A CASE STUDY

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ABSTRACT

This paper is the continuation of a previous one where the vacuum cleaner was used as an academic example for teaching Conceptual Design. In this paper the example is extended by including the use of the Linearity Theorem as a screening tool that allows the student to select the best configuration. The paper shows several contradictions that arise when students have to face design problems with the use of Axiomatic Design in a qualitative way. In particular, ambiguities derived from the individual or sequential application of the Axioms are described. The Linearity Theorem is proposed as a powerful tool for applying both Axioms at the same time and hence for overcoming the difficulties associated to the aforementioned contradictions.

Keywords: Axiomatic Design, Education, Design Matrix, Linearity Theorem.

1 INTRODUCTION / STATE OF THE ART

Independence Axiom (A1), "maintain the independence of the functional requirements (FRs)" [Suh, 1990], leads to formulate the design matrices (DM) as the main tool for assessing designs, and hence, for guiding the decision-making process during the conceptual design.

Information Axiom (A2), "minimize information content" [Suh, 1990], leads to formulate the probability of success as the main guide for refining the search of solutions.

In the classical methodology proposed by Suh there exists an asymmetry between both axioms. The asymmetry is born because priority is given to the Independence Axiom. Indeed, the Information Axiom is used for selecting the best design among all the designs that fulfill the Independence Axiom [Suh, 1990, 2001]. Benavides [2012] also shows that it is wise to force firstly the accomplishment of A1 and secondly the accomplishment of A2 because this way facilitates the calculation of the probability of success. Therefore, the asymmetry consists in preferring a sequential application of the axioms in detriment of a parallel application of them. The preference of the sequential approach is clearly stated in this alternate statement of A2 "The best design is a functionally uncoupled design that has the minimum information content" [Suh, 1990].

In addition, during the conceptual design phase it is difficult to calculate the probability of success and thus, A2 must be applied in a qualitative way; Suh [2001] or Benavides [2012] present theorems that help the designer in this task. But in several cases, the fact of having a pure qualitative description makes the result of a sequential application of the axioms different from the result of a parallel application of the axioms. From a formal point of view, this fact resembles a contradiction in the Axiomatic Design Theory. The contradiction is born because the definition of the Best Design as the one that has a Diagonal Design Matrix, requires the application of both axioms at the same time [Suh, 1990, Benavides, 2012], but it does not include all the restrictions imposed by the second Axiom. In other words, the classical methodology for qualitatively selecting the Design Parameters (DPs) only seeks to achieve a diagonal matrix without using any other additional knowledge derived from the Information Axiom. The main reason is the difficulty to derive qualitative guidelines from the quantitative formulation of the information content.

In this paper we use the same lecture structure that was given by Rodriguez and Benavides [2013] for the resolution of a pedagogical case study. The main concern of that work was to help students and practitioners in the comprehension of Axiomatic Design principles and methodology as postulated by Suh [1990, 2001]. Hence, the lecture structure was focused on the resolution of a challenge that must join at the same time 1) enough simplicity for allowing its resolution in two or three hours, and 2) enough difficulty for letting the difficulties of applying the methodology arise. This challenge was enunciated as "obtain the best design for a vacuum cleaner". In this work we will extend the same example with the purpose of showing how a decision-making process based on qualitative arguments leads to contradictions that are sometimes detected by the students, creating in them a feeling of lack of rigor.

Fully understanding of the implications derived from a qualitative application of the A2 obligates the educator to face the aforementioned contradictions, and also obligates him to give a solution to that concern. To achieve this objective, this paper starts with the cyclone system, which was the one selected by Rodriguez and Benavides [2013] as the best design for the vacuum-cleaner challenge, and deeps in its qualitative resolution showing how the contradiction of applying a sequential approach appears. Then, a solution based on the use of a very restrictive theorem is proposed. In this case, the theorem selected is the Linearity Theorem, which was mathematically proved by Benavides [2012]. Linearity Theorem is a necessary consequence of both Axioms at the same time; thus, if the Linearity Theorem is not satisfied, at least one of the Axioms is not satisfied (normally, for uncoupled designs, it will be the Information Axiom). Therefore, what we say to our students is that it is wise to use the Linearity Theorem because in other case the fulfillment of the Axioms is in serious jeopardize, even more when only qualitative arguments are used in the design process. Paper shows how this qualitative argumentation is delivered to the students and how the contradictions appear. Finally the paper shows how the aforementioned contradictions are removed because the theorem forces a stronger use of both axioms during the qualitative argumentation, solving the contradiction and producing a unique response to the proposed challenge.

2 PROPOSED STRUCTURE

The steps for conducting pedagogical lectures are [Rodriguez and Benavides, 2013]:

1. **Quantitative formulation of the design problem.**
   a. Challenge definition.
   b. Selection of the minimum number of independent FR in a neutral solution environment.
   c. Establishment of constraints.
   d. Definition of FRs in terms of measurable variables
2. **Description of the solution through its main DPs.**
   e. Writing of the design equations (physical laws).
   f. Identification of DPs.
   g. Establishment of new constraints derived from the DPs.
3. **Writing of the design matrix.**
4. **Analysis with the use of the Independence Axiom.**
5. **Analysis with the use of the Information Axiom.**
6. **Analysis with the use of the Linearity Theorem. Selection of DPs (this may imply to create new DPs and a new physical solution).**
   h. Forcing the fulfillment of the Linearity Theorem.
   i. Establishment of future improvements.
   j. Outline new challenges.

This lecture structure is the same described by Rodriguez and Benavides [2013], and it is in turn based on the methodological steps described by Suh [1990, 2001]. However in this paper, the sixth step is completed with the Linearity Theorem.

Linearity Theorem (statement 1): Linear designs are better than non-linear designs.

The proof of this theorem requires both Axioms [Benavides, 2012] and is out of the scope of the pedagogical lecture presented in this paper. For practical uses, mainly in the 6th point of the lecture, two alternative statements of the Linearity Theorem are explained to the students:

Linearity Theorem (statement 2): The ideal design matrix is uncoupled and constant.

Linearity Theorem (statement 3): When two different DPs can be used for satisfying one FRs, the best DP (the DP that must be chosen by the designer) is the most linear one.

Since the proof of the Linearity Theorem uses the Axioms, the fact of forcing its accomplishment during the conceptual design is a necessary condition (but not sufficient) for the accomplishment of the Axioms. The difference between using the theorem or not appears because the Linearity Theorem forces a stronger definition of the ideal design: The best design must have (a null information content and) a diagonal design matrix and a constant design matrix. This definition imposes that all the elements in the design matrix of the ideal design must be constant, what is a direct consequence of the Linearity Theorem [Benavides, 2012]. Note that, in general, the qualitative formulation of the probability of success avoids checking if the information content is null (this fact is even more patent when students are involved because a teaching lecture tends to be more qualitative than quantitative); i.e. in a scenario where the information content cannot be calculated because there are only qualitative argumentations, the information content is not a practical part of the ideal design definition, and for that reason we put that part in brackets in the classical definition: The best design must have (a null information content and) a diagonal design matrix; which is a version of the alternate statement of A2 given by Suh: "the best design is functionally uncoupled and contains minimum information content". (Note that it's an alternate statement and not a definition.) Note also that the new definition of the ideal design, given by the statement 3 in this paper, comes from both Axioms at the same time and, for that reason, makes a more intensive use of A2 than the classical one. To have a more exigent definition of the ideal design helps to remove or solve some of the contradictions that appears due to not calculate the information content and use qualitative argumentations instead. This problem is more severe at preliminary stages of conceptual design, and hence, it is where it is worthier to use the Linearity Theorem.

2.1 QUANTITATIVE FORMULATION OF THE DESIGN PROBLEM

2.1.1 CHALLENGE DEFINITION

Since the challenge "Analyze two different technologies (porous filter and centrifugal separation) for filtering dust particles when vacuum cleaning. Identify their main dependences and select the best solution according to Axiomatic Design." was solved in a previous work [Rodriguez and Benavides, 2013], the students know that the best solution is the one based on the cyclone physics. For that reason, the new challenge for the new lecture is: "Analyze a vacuum cleaner that uses the technique of centrifugal separation for filtering dust particles. Identify their main dependences and improve the solution according to Axiomatic Design."

2.1.2 SELECTION OF THE MINIMUM NUMBER OF INDEPENDENT FRS IN A NEUTRAL SOLUTION ENVIRONMENT

FR1: Clean-up dust particles.

FR2: Retain dust particles.

FR3: Operate during a fixed period of time.

The concept of direct independence [Benavides, 2012] is used to realize that the needs stated in FR1, FR2 and FR3 are functional requirements because they represent, in a neutral solution environment, independent concepts.

2.1.3 ESTABLISHMENT OF CONSTRAINTS

The main set of constraints that usually appear are: minimum cost, maximum energetic efficiency, easily of use. But also the number of particles collected by unit of time. For the purpose of this lecture they are not relevant.

2.1.4 DEFINITION OF FRS IN TERMS OF MEASUREABLE VARIABLES

FR1: Clean-up dust particles: u_1

FR1 represents the functionality of cleaning-up particles, which might be represented by the speed of the air that must remove the dust particles from the floor. Let u_1 be this speed.

FR2: Retain dust particles: $d_{\min}$

FR2 represents the functionality of separating all the particles that have a size bigger than $d_{\min}$.

FR3: Operate during a fixed period of time: $t_{\max}$

FR3 represent the maximum period of time that the customer expects to operate the device.

2.2 DESCRIPTION OF THE SOLUTION THROUGH ITS MAIN DPS

We will construct the design matrix using directly the physical model that relates the FRs to the DPs for the centrifugal based solution. This physical model is based on the one described by Rodriguez and Benavides [2013]. For the sake of clarity, we will repeat here the main laws that lead to relate the final value of the FRs to a given set of values of the DPs.

2.2.1 WRITING OF THE DESIGN EQUATION (PHYSICAL LAWS)

We will assume that the air can be modelled as an incompressible fluid. Under this assumption, the mass flow rate $\dot{m}$ through the system can be modeled as

$$\dot{m} = \rho_0 A_1 u_1 \tag{1}$$

where ρ_0 is the air density at room conditions, and A_1 is the cross sectional area of the inlet pipe (see Fig. 1).

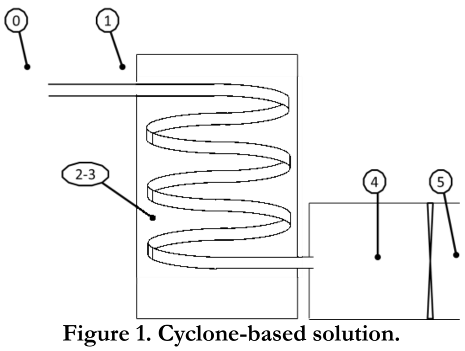

Figure 1. Cyclone-based solution.

The second physical law considers that there is an electrical motor that converts electrical energy into kinetic energy. If we assume that the available power after the conversion is $\dot{W}$, the mechanical energy injected into the air is given by:

$$\dot{W} = \dot{m}\frac{u_1^2}{2} \tag{2}$$

The combination of Eqs. (1) and (2) lead to the physical law for FR1:

$$u_1 = \frac{\dot{m}}{\rho_0 A_1} = \sqrt[3]{\frac{2\dot{W}}{\rho_0 A_1}} \tag{3}$$

The next physical law to take into account is the one that describes the centrifugal force in the cyclone. The cyclone comprises a stream tube which follows a helical stream line with a characteristic radius R and N_c turns. The differential equation that describes the radial displacement of a dust particle inside the cyclone is:

$$\frac{4}{3}\pi\left(\frac{d_d}{2}\right)^3 \rho_d \ddot{x} = \frac{4}{3}\pi\left(\frac{d_d}{2}\right)^3 \rho_d \frac{u_1^2}{R} - \frac{1}{2}\rho_0 \dot{x}^2 c_d \left(\frac{\pi d_d^2}{4}\right) \tag{4}$$

For large particles or for low radial speeds the following inequality holds:

$$\frac{\frac{1}{2}\rho_0 \dot{x}^2 c_d \frac{\pi d_d^2}{4}}{\frac{4}{3}\pi\left(\frac{d_d}{2}\right)^3 \rho_d \frac{u_1^2}{R}} << 1 \tag{5}$$

Under this condition Eq. (4) yields to:

$$\ddot{x} = \frac{u_1^2}{R} \tag{6}$$

This equation can be integrated to obtain:

$$\dot{x} = \frac{u_1^2}{R} t \tag{7}$$

$$x = \frac{1}{2}\frac{u_1^2}{R} t^2 \tag{8}$$

The time spent by the particle inside the cyclone is:

$$t = \frac{2\pi R N_c}{u_1} \tag{9}$$

Taking into account Eqs. (5) to (9), we can write the condition for neglecting the aerodynamic forces:

$$d_p >> 3\pi^2 N_c^2 c_d \frac{\rho_0}{\rho_d} R \tag{10}$$

A particle will escape from the cyclone towards the container when $x \geq d_1$, where d_1 represents the diameter of the tube (note that $A_1 = \pi d_1^2 / 4$). Thus a particle will reach the container if the following inequality is satisfied:

$$N_c \geq \frac{1}{\pi}\sqrt{\frac{d_1}{2R}} \tag{11}$$

Since $d_1 << R$ and $N_c > 1$, the previous inequality is always satisfied. Thus, all the particles that have a large size (defined by Eq. (10)) are always retained with independence of the value of the rest of the DPs. As a consequence, only if $d_{\min}$ is less than the value $3\pi^2 N_c^2 c_d R \rho_0 / \rho_d$, FR2 is not satisfied. Hence, it is convenient to simplify the Eq. (4) for the case of having particles with a diameter much lower than that. In that situation, the aerodynamic force becomes dominant and the radial velocity becomes constant as stated by:

$$\dot{x} = \sqrt{\frac{3}{4c_d}\frac{\rho_d}{\rho_d}d_d\frac{R}{u_1^2}} \tag{12}$$

This equation can be integrated to obtain:

$$x = \sqrt{\frac{3}{4c_d}\frac{\rho_d}{\rho_d}d_d\frac{R}{u_1^2}}t \tag{13}$$

Taking into account Eq. (9), the minimum size of the dust particles retained into the container is obtained from the condition $x(t) = d_1$, which yields to the design equation for FR2:

$$d_{\min} = \frac{3c_d}{4\pi^3}\frac{\rho_0}{\rho_d}\frac{A_1}{RN_c^2} \tag{14}$$

The last physical law relates the volume of dust collected during a period of time to the available volume in the chamber 2-3. Obviously, this time will depend on the density of dust particles per unit of volume n. The operational time is calculated when the whole dust container, whose volume is V_{23}, is full:

$$t_{\max} = \frac{\dfrac{V_{23}}{\dfrac{4}{3}\pi\left(\dfrac{d_d}{2}\right)^3}}{n\dfrac{\dot{m}}{\rho_0}} = \frac{3V_{23}}{2\pi d_d^3 n\sqrt[3]{2\dot{W}A_1^2}} \tag{15}$$

2.2.2 IDENTIFICATION OF DPS

The DPs derived from design equations Eqs. (3), (14) and (15) are A_1, V_{23}, N_c, R, and $\dot{W}$. (At this point, students are advised of having a redundant design with three FRs and five DPs). Which three DPs are the best ones?

2.2.3 ESTABLISHMENT OF NEW CONSTRAINTS DERIVED FROM THE DPS.

One option for selecting only three DPs is to collect several of them just in the same term. For example, we could use $\dot{W}$ or A_1 for assuring FR1 because both appear, combined as the main parameter $\dot{W}/A_1$, in Eq. (3). When only the main DPs are used to describe the solution, the physical formulation can be expressed in a qualitative DM as follows:

$$\begin{pmatrix}\text{Clean-up dust particles}\\ \text{Retain dust particles}\\ \text{Operate a long time}\end{pmatrix} = \begin{pmatrix}X & 0 & 0\\ X & X & 0\\ X & x & X\end{pmatrix}\begin{pmatrix}\text{Vacuum }(\dot{W}/A_1)\\ \text{Cyclone }(RN_c^2)\\ \text{Container capacity}(V_{23})\end{pmatrix} \tag{16}$$

The lower case x in the matrix reminds that there is a weak dependency in that term: the less the minimum size of the collected dust is, the larger the number of dust particles per unit of volume collected in the reservoir. However since this variation affects to the particles with minimum volume, we will neglect this effect, i.e. we will consider $\partial n/\partial(RN_c^2) \approx 0$.

At this point, we explain that the DM in Eq. (16) is not linear due to the dependence of FR2 and FR3 with A_1. In addition, we remark in class that the groups of DPs made in Eq. (16) are not unique, and hence there exist some degree of arbitrariness in that Design Matrix. In matrix (16), DP1 was chosen as the specific power installed in the system because it is a quite engineering parameter that measures the amount of power per unit of system size, and hence, it allows the engineer to compare systems of different sizes. It affects to FR2 because if DP1 changes due to a variation of A_1 Eq. (14) shows that FR2 varies too. However, if the inlet area is removed from DP1 because it is not considered a main DP any more, the physical formulation also responds to the following DM:

$$\begin{pmatrix}\text{Clean-up dust particles}\\ \text{Retain dust particles}\\ \text{Operate a long time}\end{pmatrix} = \begin{pmatrix}X & 0 & 0\\ 0 & X & 0\\ X & 0 & X\end{pmatrix}\begin{pmatrix}\text{Vacuum }(\dot{W})\\ \text{Cyclone }(RN_c^2)\\ \text{Container capacity}(V_{23})\end{pmatrix} \tag{17}$$

Both matrices show that the design is uncoupled for FR1 and FR2. However, because the former has a less number of off-diagonal elements, the second one is closer to the ideal design. This difference comes from reducing the number of DPs to the number of FRs such as the ideal design requires. This reduction is, in fact, made by imposing design constraints in the DPs not selected [Suh, 1990]. Therefore, depending on how the designer defines the constraints that affect the DPs, several design matrix are obtained. This resembles an inconsistency in the qualitative application of the Axiomatic Theory because the first design matrix induces us to think that using the specific power as a design parameter is worse than using just the power for a frozen value of the area, but engineers have well-founded arguments for preferring the specific power. This incoherence is quite important from the academic point of view because it produces a negative impact in the students that realize that question. They perceive the existence of different DMs for the same problem as an arbitrariness in the decision making process, and hence, they tend to reject the Axiomatic Design Theory for not considering it a scientific or well-founded methodology when a tool to select the most accurate DPs. In our lectures we solve this problem saying that all the variables that can be controlled by the designer are DPs, but that only a few number of them are the best ones.

2.3 WRITING OF THE DESIGN MATRIX

Our approach for removing the flaw addressed before is to accept that all the physical variables are susceptible of being DPs and hence, all of them must be considered main parameters. Thus, according to the design equations written before, the resultant design matrix can be written as follows:

$$\frac{\partial u_1}{\partial \dot{W}} \neq 0; \frac{\partial u_1}{\partial A_1} \neq 0; \frac{\partial u_1}{\partial R} = \frac{\partial u_1}{\partial N_c} = \frac{\partial u_1}{\partial V_{23}} = 0 \tag{18}$$

$$\frac{\partial d_{\min}}{\partial \dot{W}} = \frac{\partial d_{\min}}{\partial A_1} = \frac{\partial d_{\min}}{\partial V_{23}} = 0; \frac{\partial d_{\min}}{\partial R} \neq 0; \frac{\partial d_{\min}}{\partial N_c} \neq 0 \tag{19}$$

$$\frac{\partial t_{\max}}{\partial \dot{W}} \neq 0; \frac{\partial t_{\max}}{\partial A_1} \neq 0; \frac{\partial t_{\max}}{\partial V_{23}} \neq 0; \frac{\partial t_{\max}}{\partial R} = \frac{\partial t_{\max}}{\partial N_c} = 0 \tag{20}$$

Resulting in the following design matrix:

$$\begin{pmatrix} \Delta u_1 \\ \Delta d_{\min} \\ \Delta t_{\max} \end{pmatrix} = \begin{pmatrix} \frac{\partial u_1}{\partial \dot{W}} & \frac{\partial u_1}{\partial A_1} & 0 & 0 & 0 \\ 0 & \frac{\partial d_{\min}}{\partial A_1} & \frac{\partial d_{\min}}{\partial R} & \frac{\partial d_{\min}}{\partial N_c} & 0 \\ \frac{\partial t_{\max}}{\partial \dot{W}} & \frac{\partial t_{\max}}{\partial A_1} & 0 & 0 & \frac{\partial t_{\max}}{\partial V_{23}} \end{pmatrix} \begin{pmatrix} \Delta \dot{W} \\ \Delta A_1 \\ \Delta R \\ \Delta N_c \\ \Delta V_{23} \end{pmatrix} \tag{21}$$

At this point, with the full formulation of the DM, the student realizes that 1) the design is coupled because A_1 affects to all the FRs, and 2) the design is redundant because there are more DPs than FRs. Both are unexpected events for a concerned student because it strongly contrasts with the classical formulation given by the DM in Eq. (17). Now the contradiction is even more apparent because the new DM (with the full set of available DPs) separates even more the design matrix from the ideal one. Thus, the previous inconsistency, far from disappearing, becomes stronger, which transmits again the impression of methodological weakness when identifying the most adequate DPs.

2.4 ANALYSIS WITH THE USE OF THE INDEPENDENCE AXIOM

The classical approach to solve this ambiguity consists on freezing the variable that does not satisfy the Independence Axiom the most and select the DPs from the rest of variables [Suh, 1990]. Therefore, the straightforward selection derived from the application of the Independence Axiom is, for example, the following one (which resembles the structure of the DM in Eq. 17)

$$\begin{pmatrix} \Delta u_1 \\ \Delta d_{\min} \\ \Delta t_{\max} \end{pmatrix} = \begin{pmatrix} \frac{\partial u_1}{\partial \dot{W}} & 0 & 0 \\ 0 & \frac{\partial d_{\min}}{\partial R} & 0 \\ \frac{\partial t_{\max}}{\partial \dot{W}} & 0 & \frac{\partial t_{\max}}{\partial V_{23}} \end{pmatrix} \begin{pmatrix} \Delta \dot{W} \\ \Delta R \\ \Delta V_{23} \end{pmatrix} \tag{22}$$

However, this approximation has the problem of applying exclusively the Independence Axiom, giving to the Information Axiom a secondary role.

2.5 ANALYSIS WITH THE USE OF THE INFORMATION AXIOM

According to the Axiomatic Design Theory the Information Axiom is applied after the Independence Axiom for selecting the best design among all the ones that fulfill the Independence Axiom. In this case, there is another possibility that could be best than the previous one. It is the following,

$$\begin{pmatrix} \Delta u_1 \\ \Delta d_{\min} \\ \Delta t_{\max} \end{pmatrix} = \begin{pmatrix} \frac{\partial u_1}{\partial \dot{W}} & 0 & 0 \\ 0 & \frac{\partial d_{\min}}{\partial N_c} & 0 \\ \frac{\partial t_{\max}}{\partial \dot{W}} & 0 & \frac{\partial t_{\max}}{\partial V_{23}} \end{pmatrix} \begin{pmatrix} \Delta \dot{W} \\ \Delta N_c \\ \Delta V_{23} \end{pmatrix} \tag{23}$$

However, in a qualitative formulation where the information content cannot be obtained, any selection between (22) and (23) appears again as an arbitrariness.

Since the Axiomatic Design postulates an objective definition of the best design, any arbitrariness in the method for obtaining it appears as a contradiction of the theory. We propose to fix this problem with the use of the Linearity Theorem.

2.6 ANALYSIS WITH THE USE OF THE LINEARITY THEOREM

The quantitative description plus the Linearity Theorem allow us to focus the students on selecting DPs or on creating new ones, if the available DPs do not fulfil the Linearity Theorem.

We think that making the design decisions on applying both Axioms at the same time is more powerful than apply them individually or sequentially. For that reason, our approach lies on explaining that it gives more value to force the solution to satisfy a theorem that comes from both axioms at the same time than to apply only one axiom (or to apply a reduced definition of the best design); in general, we assume that it gives more value to use a stronger definition of the ideal design: If it is not linear, it is not the best design.

Since the Linearity Theorem leads to a stronger definition of the best design, we encourage our students to use it for selecting the best set of DPs. In the case of the FR2, Eq. (14) tell us that FR2 has the most linear dependency with variable A_1, neither with R nor N_c. For that reason, A_1 should be chosen as the DP2; however this decision produces a contradiction with the theory itself because it leads to a coupled design that does not satisfy the Independence Axiom (and because its relation with FR1 is not linear). The removal of this contradiction is presented to the students as a new challenge. At this point we induce students to think of how this solution could be improved. More specifically, they are asked about how the dependency between u_1 and $d_{\min}$ due to A_1 could be removed: students are able to focus A_1 for the design improvement by using Linearity Theorem.

2.6.1 PROPOSING UNCOUPLED SOLUTIONS AND OUTLINE NEW CHALLENGES

The previous challenge exposes students to a creative problem with the following hints: 1) due to the Linearity Theorem, they know that *area* is an important DP, and 2) due to the Independence Axiom, they know that A_1 is a bad DP. One solution to this innovative/creative challenge is to add more *areas* as DPs: one possibility that can be explored is adding the inlet diameter A_0 as a new design parameter. This addition changes the design in Fig. 1, by adding the inlet diameter as a new design parameter, into the new solution presented in Fig. 2, where the cyclone diameter is different from the inlet diameter.

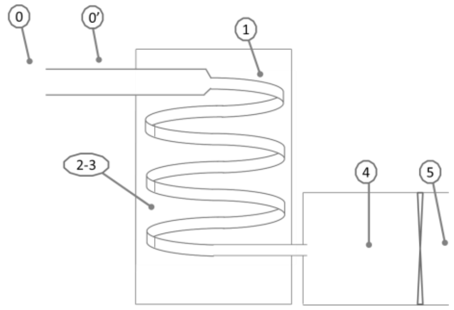

Figure 2. Cyclone-based solution with decoupled air speeds.

In this new solution a new DP has been added to the system in order to decouple the inlet speed from the cyclone speed. The new equations are

$$u_{0'} = \frac{\dot{m}}{\rho_0 A_{0'}} = \sqrt[3]{\frac{2\dot{W}}{\rho_0 A_{0'}}} \tag{24}$$

$$d_{\min} = \frac{3c_d}{16\pi^2}\frac{\rho_0}{\rho_d}\frac{A_1}{RN_c^2} \tag{25}$$

$$t_{\max} = \frac{3V_{23}}{2\pi d_d^3 n \sqrt[3]{2\dot{W}A_{0'}^2}} \tag{26}$$

And the DM derived from them is

$$\begin{pmatrix} \Delta u_{0'} \\ \Delta d_{\min} \\ \Delta t_{\max} \end{pmatrix} = \begin{pmatrix} \frac{\partial u_{0'}}{\partial \dot{W}} & \frac{\partial u_{0'}}{\partial A_{0'}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial d_{\min}}{\partial A_1} & \frac{\partial d_{\min}}{\partial R} & \frac{\partial d_{\min}}{\partial N_c} & 0 \\ \frac{\partial t_{\max}}{\partial \dot{W}} & \frac{\partial t_{\max}}{\partial A_{0'}} & 0 & 0 & 0 & \frac{\partial t_{\max}}{\partial V_{23}} \end{pmatrix} \begin{pmatrix} \Delta \dot{W} \\ \Delta A_{0'} \\ \Delta A_1 \\ \Delta R \\ \Delta N_c \\ \Delta V_{23} \end{pmatrix} \tag{27}$$

This allows us to write the following design matrix

$$\begin{pmatrix} \text{Clean-up dust particles} \\ \text{Retain dust particles} \\ \text{Operate a long time} \end{pmatrix} = \begin{pmatrix} X & 0 & 0 \\ 0 & X & 0 \\ X & 0 & X \end{pmatrix} \begin{pmatrix} \text{Vacuum } (\dot{W} / A_{0'}) \\ \text{Cyclone } (A_1 / RN_c^2) \\ \text{Container capacity}(V_{23}) \end{pmatrix} \tag{28}$$

This shows that although DM in Eq. (17) is formally similar to the DM in Eq. (28), the physical solutions are not. Therefore, the contradictions coming from the arbitrariness were due to forget to implement a new design parameter. This is quite interesting because in this problem the number of DPs was already larger than the number of FRs. However, this possibility still can use A_1 as DP2 and $\dot{W}$ as DP1, while still keeping the design matrix for FR1 and FR2 diagonal.

3 CONCLUSION

This paper explores some contradictions and arbitrariness that can appear when Axiomatic Design is used in a qualitative way, particularly, when qualitative examples are used for teaching it to students and the ideal design is exclusively based on the functional independence. It is proposed that using a definition of the best design as strong as possible mitigate these problems. In this paper, the stronger definition is "the best design is the one that has a constant and diagonal Design Matrix" which arises from the Linearity Theorem. In particular, it has been shown that this definition of the Best Design fixes that problem for the case study of the cyclone vacuum cleaner, and it has been shown that the solution requires increasing the number of physical variables.

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