Proceedings of ICAD2014
The Eighth International Conference on Axiomatic Design
Campus de Caparica – September 24-26, 2014
ICAD-2014-24

# AN AD OPTIMIZED METHOD FOR THE STRUCTURAL ANALYSIS OF PRESSURE VESSELS

**Andrea Girgenti**
a.girgenti@unimarconi.it
Department of Technologies and Innovation Development
University Guglielmo Marconi
Via Plinio, 44 - 00193 Roma –Italy

**Cosimo Monti**
c.monti@unimarconi.it
Department of Technologies and Innovation Development
University Guglielmo Marconi
Via Plinio, 44 - 00193 Roma –Italy

Alessandro Giorgetti
a.giorgetti@unimarconi.it
Department of Technologies and Innovation Development
University Guglielmo Marconi
Via Plinio, 44 - 00193 Roma –Italy

Paolo Citti
p.citti@unimarconi.it
Department of Technologies and Innovation Development
University Guglielmo Marconi
via Plinio, 44 - 00193 Roma –Italy

## ABSTRACT

The Axiomatic Design theory allows finding the right solutions in order to respond to functional needs of a product. Adopting the best solution is important in order to avoid a non-optimal design that can result in losses of time and money.

Axiomatic Design theory (or simpler AD theory) may concern not just the design of a product but it can also deal with defining processes or tools.

The usage of the most efficient and effective tools is a more and more important issue in engineering activities in order to achieve results with short times and optimized resources and funds. From this point of view the Axiomatic Design theory can be used in order to select the proper calculation method that permits to achieve these goals. Pressure vessels are quite common parts in several machines or plants, usually they present round bottoms but sometimes a flat bottom is required to minimize their sizes.

The scope of this paper is to introduce an optimized AD calculation method in order to perform structural analysis of a pressure vessel with a lower planeside and several reinforcements.

Two methods may be used to solve the structural issue. They will be introduced in the following passages. Their design matrices, according with the first axiom, will be found and on the basis of these matrices a comparison between these approaches will be carried out.

**Keywords:** Stress and strain evaluation, pressure vessel, energy method, reinforced flat bottom.

## 1 INTRODUCTION

Choosing the best solution, according to the axiomatic design theory, is increasingly important in modern engineering activities to reach maximum performance in terms of timesaving and economical convenience. The theory [*Suh*, 1990] and [*Suh*, 2001] is not valid just to perform the best design related to goods or services but it could be successfully applied also at physical models and calculation methods which are common in engineering practice. Choosing the wrong solution could lead to loss of time and higher costs for whatever these applications, not depending on their kind. The usage of calculation methods and tools, which permit to minimize the effort in order to simulate mechanical systems behaviour, is deeply important to achieve time reduction in several engineering issues and allows being able to absorb eventual delays in production or in other functions of the company. In this paper it will be shown how engineers can use Axiomatic Design in order to select and use the best model and calculation method to achieve valid results and solve some problems in designing goods with efficiency and efficacy. In this paper two different approaches will be described in order to solve the issue concerning the determination of stresses in a pressure vessel. Both of them will be compared from the point of view of the AD theory.

## 2 SCOPE AND USED APPROACH

This paper will show an alternative calculation approach to solve a structural problem, finding both stresses and strains at the bottom of a pressure vessel. This model will be optimized from the point of view of Axiomatic Design theory in respect of a more classical method. The design matrices for both the approaches will be drawn and their compliance with the best condition, according with the first axiom, will be examined.

Both the two methods may be used to find stresses and strains of a vessel which is steel made and has some welded reinforcements under the lower plane side (Figure 1). The found values for stresses are useful to be compared with the maximum one for the material according with the most common failure criteria (Von Mises's or Tresca's).

A case study for the validation of the proposed method is going to be presented in this paper. Both the alternative method and the FEM analysis will be applied to a pressure vessel with a cylindrical sidewall and a welded flat bottom. The vessel is pressurized inside and its upper side is tight closed by a cap. A hydrostatic pressure is assumed to be present within the vessel. Five beams are welded under the lower flat end in order to improve both the stiffness and the strength and to avoid bending. The reinforced vessel can be seen in Figure 1.

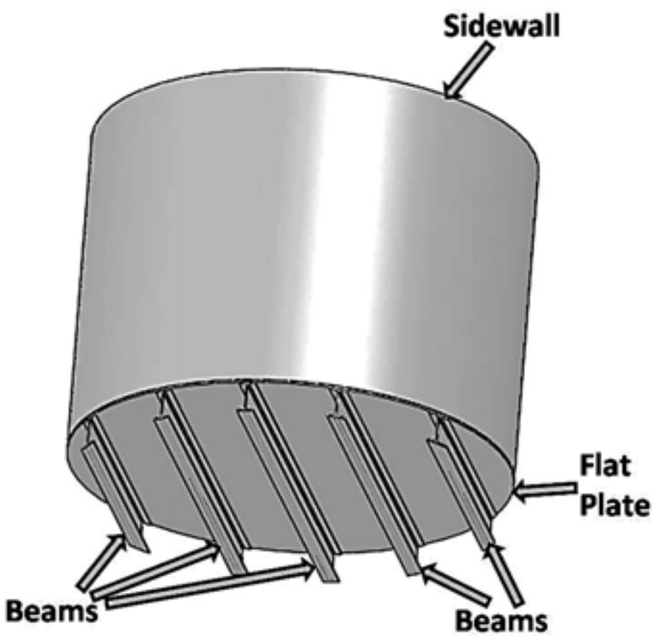


**Figure 1** Pressure vessel with reinforced fl**at bottom.**

Before taking into account how to model and find the stresses in the reinforced lower end, the issue of the structural strength under load for the whole vessel and its parts has been analyzed from an axiomatic design point of view.

## 2.1 DECOMPOSITION OF THE PROBLEM CONCERNING DETERMINATION OF STRESSES.

Obviously the whole vessel has to resist to the internal pressure, so the complete map of the requested checks can be defined by decomposing the highest-level functional requirement (FR) and the corresponding design parameter (DP), creating hierarchies of FRs, and DPs. This is done through zigzagging among the physical and functional domains, according with the Thompson's notes [Thompson, 2013].

The main functional requirement (FR) at the higher level expresses what the vessel should do. According with the axiomatic theory several main FRs could be found by referring to specific regulations about this kind of application.

This case deals with one FR design since the main FR is "To resist under the stresses due to pressure" and it is common for both the methods since it is a general requirement at this level of decomposition. This is valid for the whole vessel, not depending by how the calculation of stresses in the reinforced bottom is achieved. A way to achieve this main FR is obviously the right "sizing of all parts", that is DP1. The main FR is further split into several FRs at the second layer of zigzagging depending on where the vessel shall resist; so the FR1.1 is "to resist the lateral pressure, FR1.2 is "to resist the pressure on the cap" and FR1.3 is "to resist the pressure at the bottom side". The DPs that respond to the stated FRs are: "stresses in the sidewall" (DP1.1), "stresses in the cap" (DP1.2) and "stresses in the bottom area" (DP1.3). Figure 2 shows the design matrix for these upper layers of mapping which links requested checks (FRs) and corresponding design parameters (DPs).

| | DP1 | DP1.1 | DP1.2 | DP1.3 |
|---|---|---|---|---|
| FR1 | X | | | |
| FR1.1 | | X | | |
| FR1.2 | | | X | |
| FR1.3 | | | | X |

**Figure 2 Design matrix concerning first checks and** upper layers decomposition.

This design matrix links through its diagonal the first checks (or functional requirements) to the corresponding design parameters. This means that each check can be solved independently from the others.

At a lower layer, continuing the decomposition, single formulas can be found. These link geometries to the stresses values at least for FR1.1 and FR1.2. Finding internal stresses in the sidewall and in the cap is quite easy using models that are dealt in literature [*Jawad*, 1994], [*Moss,* 2004] [*Nerli*, 2005], considering the hypothesis of small thickness wall because of the big ratio between thickness and diameter of the vessel. Referring to Figure 3 the values for tangential stresses $\sigma_t$, radial stresses $\sigma_r$ and axial stresses $\sigma_a$are those shown through (1):

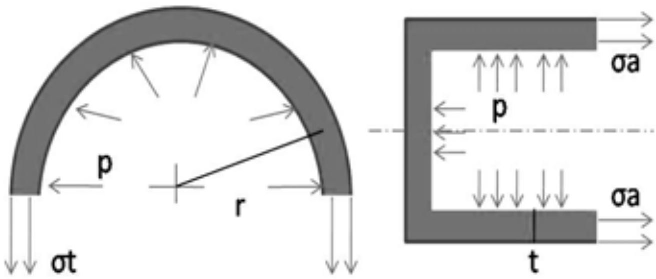


**Figure 3 Pressure** vessels theory small thickness walls.

$$\sigma_t = \frac{p \cdot r}{t} \qquad \sigma_a = \frac{p \cdot r}{2 \cdot t} \qquad \sigma_r = 0 \quad (1)$$

The cap and the sidewall are solved quite easily through the application of the classical theories of pressurized vessels. On the other hand, computing the contributes of the five beams on the overall strength of the bottom is a bit more complicated since we have to solve a hyperstatic system with several unknown variables. Due to this, finding the stresses and the strains in the bottom of the vessel could be expensive in terms of time and resources. An alternative is represented by an energy criterion which permits to compute stresses through the width of an equivalent system whose the elastic energy is the same as the sum of those from the single bottom and the five reinforcements. These two ways to compute stresses will be compared evaluating the agreement level to the best condition described in the first axiom of the AD theory, i.e. the decoupled design.

 

The algorithms are adopted as functional requirements since they express mathematical functions which link input and output variables. Each algorithm has to be deterministically solved; therefore it sets what input values are needed in order to find outputs. These FRs are different for all methods since they are the requested algorithms to solve the problem. Also the main DP is split in several lower DPs: they are all the requested variables to solve the algorithms.

As previously stated, the two methods are diversified by how they achieve these functions in the lower layers of zigzagging.

## 2.2 FIRST METHOD: MODELING OF REINFORCED BOTTOM LIKE AN HYPERSTATIC SYSTEM

The calculation of stresses in the lower end which is composed by a round flat plate, reinforced through welded beams, is quite difficult. A cross section of the bottom of the vessel and of the welds is shown in Figure 4. The calculations about the welds are omitted because they are well present in literature [*Caffè*, 2012]; therefore they can be executed apart.

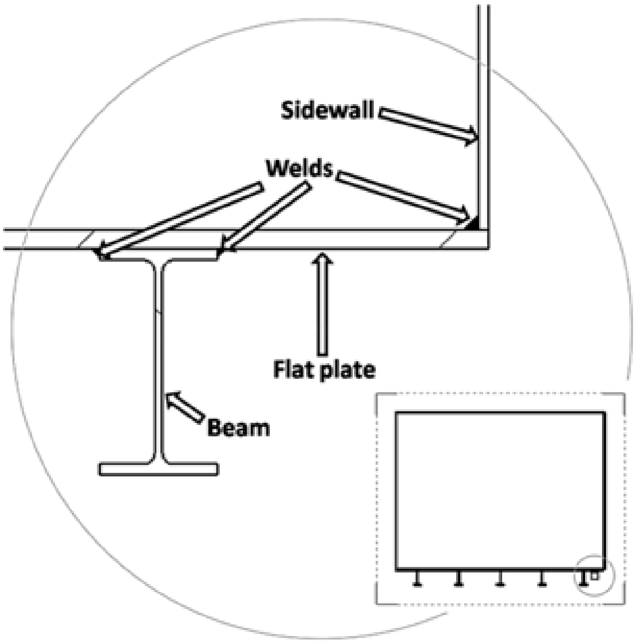


**Figure 4** Detail of a section of the bottom.

The welded beams are used in order to enhance the strength and stiffness of the flat end of the vessel. These five reinforcements, which are equally spaced, have the same cross section but different lengths to follow the perimeter of the lower round shaped plate.
Due to this assumption the flat plate and these beams may be considered as a standalone structure which is welded to the sidewall. A way to reduce the 3D problem in a 2D simpler case is achieved through considering of a narrow diametric slice of this standalone part, composed by the plate and the five beams. This passage is possible because of the specific boundary condition, i. e. the internal pressure that is uniformly distributed on the surface of the plate. The result is a structure that is loaded by a homogeneous pressure distribution, whose moments of inertia of beams are calculated in the original cross section. In Figure 5 a scheme of this representation can be seen.

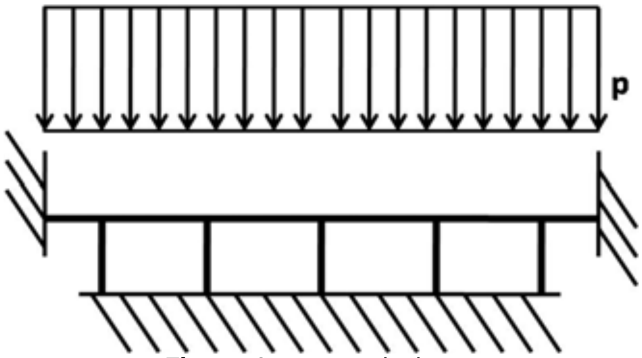


**Figure 5** 2D structural scheme.

This is clearly hyperstatic and quite complicated to solve, so it has usually calculated through FEM analysis. This 2D hyperstatic system has three degrees of freedom since it can be considered as a single structure. The authors suppose it has seven fixed joints. They are constraints that eliminate three degrees of freedom for each one. The degrees of hyperstaticity are counted through the difference between the degrees of constraints and the degrees of freedom. In this case the degrees of constraints result twenty one while the degrees of hyperstaticity are eighteen.
The authors assume that the functional requirements are satisfied by DPs; therefore, assuming a right handed orthonormal reference system whose axles are "x", "y" and "z" and the centre is "O", it allows defining the FRs. FR1.3.1 is the sum of all external forces along x axle which is equal to zero, FR1.3.2 is the sum of all external forces along y angle which is equal to zero and FR1.3.3 is the sum of all external moments along z axle which is equal to zero. These three equations are written below as the system (2):

$$\begin{cases} \sum F_x^{ext} = 0 & FR1.3.1 \\ \sum F_y^{ext} = 0 & FR1.3.2 \\ \sum M_z^{ext} = 0 & FR1.3.3 \end{cases} \tag{2}$$

The DPs are assumed to be the variables which appear in the three equations, therefore there are twenty one DPs numbered from DP1.3.1 to DP1.3.2. Since the three equations are not linearly independent all DPs will be present in every equation. The design matrix takes into account these considerations (Table 1).

The third layer of decomposition is represented through a complex matrix because equations are not linearly independent. To solve these equations and find the stresses in the bottom of the vessel, the hyperstatic system can be manually calculated; otherwise its behaviour can be simulated through a FEM analysis.

 

| | FR1 | FR1.1 | FR1.2 | FR1.3 | FR1.3.1 | FR1.3.2 | FR1.3.3 |
|---|---|---|---|---|---|---|---|
| DP1 | X | | | | | | |
| DP1.1 | | X | | | | | |
| DP1.2 | | | X | | | | |
| DP1.3 | | | | X | | | |
| DP1.3.1 | | | | | X | X | X |
| DP1.3.2 | | | | | X | X | X |
| DP1.3.3 | | | | | X | X | X |
| DP1.3.4 | | | | | X | X | X |
| DP1.3.5 | | | | | X | X | X |
| DP1.3.6 | | | | | X | X | X |
| DP1.3.7 | | | | | X | X | X |
| DP1.3.8 | | | | | X | X | X |
| DP1.3.9 | | | | | X | X | X |
| DP1.3.10 | | | | | X | X | X |
| DP1.3.11 | | | | | X | X | X |
| DP1.3.12 | | | | | X | X | X |
| DP1.3.13 | | | | | X | X | X |
| DP1.3.14 | | | | | X | X | X |
| DP1.3.15 | | | | | X | X | X |
| DP1.3.16 | | | | | X | X | X |
| DP1.3.17 | | | | | X | X | X |
| DP1.3.18 | | | | | X | X | X |
| DP1.3.19 | | | | | X | X | X |
| DP1.3.20 | | | | | X | X | X |
| DP1.3.21 | | | | | X | X | X |

Table 1 **Design matrix of the hyperstatic system**

### 2.3 SECOND METHOD: CALCULATION OF THE BOTTOM THROUGH THE ENERGY BALANCE.

The energy method avoids solving the structural analysis since it calculates the thickness of an equivalent lower plate taking into account the overall stiffness of the bottom area including the reinforcements and the plate. To achieve this simplified model, the effects of the reinforcements have been taken into account, decomposing them and neglecting which are not important. This way to look at the problem allows modelling the beams like springs, neglecting secondary effects like internal damping that is given by material. The authors suppose the lower plate and the reinforcements have the same displacements and strains since their material is the same and there are no clearances between them. Calculation about the welds is neglected and the boundary condition about pressure is the same as in paragraph 2.2. In Figure 6 a scheme of the model applied to the case study is shown.

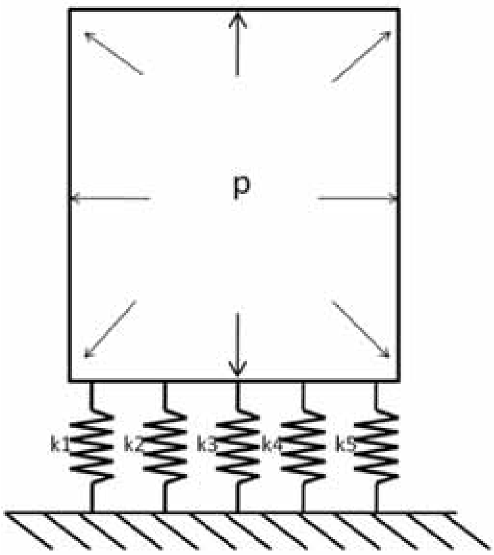


**Figure 6 Scheme of the energy** model applied to the case study.

Another hypothesis is that the stiffness coefficients for each spring, i.e. a single beam, are all the same, so the equation for the energy balance can be written as:

$$\sum_{i=1}^{5}(J_{beam-i}) + J_{plate} = J^* \qquad (3)$$

In the equation (3) $J_{beam-i}$ is the amount of the elastic energy for the i-th beam, $J_{plate}$ is the amount of the elastic energy for the flat plate and $J^*$ is the amount of the elastic energy for a dummy system which takes into account the inertia moments of the several parts. The formula for the elastic energy is well known as:

$$J = \frac{1}{2}k\Delta l^2 \qquad (4)$$

where "k" is the stiffness coefficient and "$\Delta l$" is the elongation. It can be considered that following relationships are valid for springs and in presence of a normal force:

$$F = k \cdot \Delta l \qquad (5)$$

$$F = p \cdot A \qquad (6)$$

where "A" is the normal surface in respect with the applied force. Two more formulas have to be considered:

$$p = E \cdot \varepsilon \qquad (7)$$

$$\varepsilon = \frac{\Delta l}{l_0} \qquad (8)$$

where $l_0$ is the initial length (no applied loads) of these springs or the initial thickness for the plate (named $s_0$in such case). If (5), (6), (7) and (8) are substituted in (4) it can be obtained that $J = AE\varepsilon^2\Delta l_o$ and (3) becomes:

$$\sum_{i=1}^{5}\left(\frac{1}{2}A_iE\varepsilon^2 l_0\right) + \frac{1}{2}A_{plate}E\varepsilon^2\, s_0 = \frac{1}{2}A_{plate}E\varepsilon^2 s^* \qquad (9)$$

The product $\frac{1}{2}E\varepsilon^2$ is the same for all polynomials since both the vessel and several beams are made by the same steel, according with the hypothesis, so it is easily elided from the equation above and (9) becomes:

$$l_0 \cdot \sum_{i=1}^{5} A_i + A_{plate} \cdot s_o = A_{plate} \cdot s^* \qquad (10)$$

where $s^*$ can be found using equation (10) and taking into account the moments of inertia of the system. Then the stresses can be calculated as the stresses in a round plate under a constant pressure distribution. The formulas for the calculation are present in literature [*Jawad*, 1994], [*Moss*, 2004] and [*Szilard*, 2004] and the problem is reduced just to one equation with one unknown variable.

According with the defined functional requirements and the design parameters, the two equations to calculate $s^*$ and the equivalent state of stresses $\sigma_{eq}$ are assumed as FR1.3.1

and FR1.3.2, while $s^*$ and $\sigma_{eq}$ are DP1.3.1 and DP1.3.2. The design matrix for this method is shown in Table 2.

| | DP1 | DP1.1 | DP1.2 | DP1.3 | DP1.3.1 | DP 1.3.2 |
|---|---|---|---|---|---|---|
| FR1 | X | | | | | |
| FR1.1 | | X | | | | |
| FR1.2 | | | X | | | |
| FR1.3 | | | | X | | |
| FR1.3.1 | | | | | X | |
| FR1.3.2 | | | | | | X |

Table **2 Design Matrix for the Energy Method.**

## 2.4 RESULTS AND VALIDATION OF THE MODEL.

The energy model has been applied to a case study in order to compare results with those achieved from a FEM analysis. The aim of this comparison is having a first validation of the proposed energy method that has been obtained considering AD first axiom. A pressure vessel made by steel with a flat bottom and a cylindrical sidewall has been considered for the case study. Under the flat plate five beams are welded and the hydrostatic pressure inside is 3 bar. The result of the FEM analysis for stresses is shown in Figure 7.

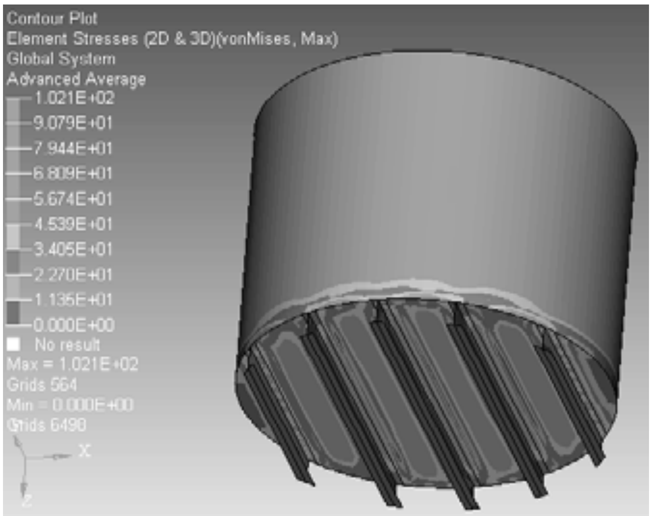


**Figure 7** Results of the FEM Analysis for stresses.

The maximum value for stresses in the bottom is almost 102 MPa. The achieved stress value from the AD optimized approach is 99,7 MPa, therefore the comparison between equivalent stresses from both the methods has shown an estimated difference next to 2.3%. This slight difference between results leads to consider valid the proposed model in order to perform a first sizing or structural check using the AD optimized method.

## 3 CONCLUSIONS

In this paper an AD optimized example for structural analysis of pressure vessels is proposed. Pressure vessels are very common in industrial applications and although the round bottom ones are often better choices, in several cases the flat bottom vessels are requested. This kind of vessels is often reinforced through beams and two approaches to calculate stresses within the parts have been presented.

Two methods to calculate stresses into the structure have been examined according to Axiomatic Design Theory: The first method is the classical approach which is achieved through the resolution of a hyperstatic system, the second one takes into account a balance among the elastic energies of the parts under pressure. This alternative model, which is optimized from an AD point of view, has been proposed in order to solve structural analysis that otherwise would be quite complicated. The diagonal matrix, which links FRs and DPs, has been drawn out for this method, resulting in a decoupled design.

A validation of the proposed model has been performed comparing the achieved results to the obtained ones from a FEM analysis. This comparison has shown how values for stresses in the reinforced flat bottom from both the approaches are quite near and the proposed model can be used in a first structural check with good level of approximation.

## 4 REFERENCES

[1] Bucalem M., Bathe K., *The Mechanics of Solids and Structures - Hierarchical Modeling and the Finite Element Solution,* Springer, 2011, ISBN 978-3540263319

[2] Caffè S., *Acciaio. Manuale tecnico per il progetto e la verifica delle strutture in acciaio e delle connessioni bullonate e saldate,* Grafill, 2012, ISBN 978-8882074999

[3] Jawad M., *Theory and Design of Plate and Shell structures,* Springer, 1994, ISBN 978-0412981814

[4] Moss D., *Pressure Vessel Design 3rd Edition,* GPP Elsevier, 2004, ISBN 978-0080524122

[5] Nerli G., *Costruzione di Macchine*, Noccioli Editore, 2005, Florence

[6] Suh N.P., *Axiomatic Design*, New York: Oxford University Press, 2001. ISBN 978-0-19-513466-7

[7] Suh N.P., *The Principles of Design*, New York: Oxford University Press, 1990. ISBN 0-19-504345-6

[8] Szilard R., *Theories and Applications of Plate Analysis: Classical Numerical and Engineering Methods*, 2004, ISBN 978-0471429890

[9] Thompson M.K., "A Classification of procedural Errors in the Definition of Functional Requirements in Axiomatic Design Theory", *The $7^{th}$ International Conference on Axiomatic Design, ICAD '13*, Worcester, Massachusetts, USA, Jun. 26-28, 2013.