AXIOMATIC DESIGN OF AN IMPROVED EGG CARTON MANUFACTURING PROCESS

Andelle Kudzal
akudzal@wpi.edu
Department of Mechanical Engineering
Worcester Polytechnic Institute
100 Institute Road
Worcester, MA, 01606, United States

Zhaolong Zhang
zzhang4@wpi.edu
Department of Mechanical Engineering
Worcester Polytechnic Institute
100 Institute Road
Worcester, MA, 01606, United States

Justin Lagassey
lagasseyj@wpi.edu
Department of Chemical Engineering
Worcester Polytechnic Institute
100 Institute Road
Worcester, MA, 01606, United States

ABSTRACT
An egg carton fabrication process was developed and evaluated using the Axiomatic Design principles. The process is improved via new approaches that could better match the functional requirements. In addition, this new egg carton design should have better protection against bacteria.

Keywords: Axiomatic design, egg carton, design matrix

1 INTRODUCTION
1.1 OBJECTIVES
The objective for this project is to decrease waste energy, reduce cost, and improve production of an egg carton by refining the manufacturing process.

1.2 RATIONALE
Eggs in the United States have a high market penetration, with 92.8% of households purchasing eggs at least once per year [Nielson, 2010]. The traditional egg carton manufacturing process is inherently wasteful. Increasing energy efficiency will reduce operating costs. Egg carton surfaces favor microbial growth and can increase the possibility of transmission of diseases to consumers [Parker et al. 2011].

1.3 STATE-OF-THE-ART
The two most popular materials for egg cartons are polystyrene and pulped paper. Compostable materials have also been used; in Japan egg cartons have been manufactured using bamboo [Pacific Research Company, 2004]. Initial scientific research has been made into food packaging made of polyactic acid derived from corn [Gross et al. 2002]. Polystyrene can be delivered in bulk as pellets ready for extrusion or can be synthesized from styrene monomer and peroxides on site [Zabanitiou et al. 2009]. Hungenberg et al. 2005. Polystyrene uses less raw material and is cheaper than pulped paper [Hocking, 2003].

Raw materials must be processed to prepare it for manufacturing. Polystyrene pellets are extruded into foam sheets. Pellets are fed into a single screw extruder where heat, foaming gas and friction are applied [Total Petrochemicals, 2013]. This causes the pellets to melt and small gas pockets to form and expand upon exiting the extruder to form the desired solid foam [Harold et al. 2007, Schuler et al. 2012].
Pulp processing requires raw recycled paper blended with water and pulp slurry is continuously stirred in a blending drum. The pulp is then sphoned off and screened [Zabanitiou et al. 2009].

Dating back to early 1950s', engineers started to make egg carton using collapsible cardboard [Gilchrist, 1953]. Semi-rigid materials, such as fiberboard or cardboard, were mainly produced to make egg cartons by folding and stretching processes [Randles, 1968]. Egg cartons from pulped paper are produced though a drying and punching processes [Lord et al. 1966, Jie, 2013]. Thermoforming of thermoplastic and punching of polystyrene is also used to make egg cartons [Pavuk et al. 1976, Pearl et al. 1971].

The purpose of printing on the egg carton is to provide production information to the customers. Brand, production date, size, weight, number of eggs, barcode and nutritional data should be printed on the cartons [Colorado Department of Agriculture, 2013]. For clear plastic cartons, information is printed on a paper label that is applied to the cartons. For paperboard cartons, information is printed directly on the carton [Carlson et al. 1991]. Flexographic printing has also been used [Hashimoto et al. 1990]. Patents have also been filed for a process for printing on both sides of the egg carton using a rotary cylinder [Provan, 1973]. Board printers can also be used to control the pressure to ensure every product is printed with same quality [Cavagna, 1990]. Laser marking system uses a laser in order to etch directly on to the egg carton requiring no physical interaction [VideoJet Technolo-gies Inc., 2012].

1.4 APPROACH
This paper presents an improved approach of designing egg carton production. The manufacturing process is improved by reducing both the production energy and time, as well as decreasing product susceptibility to bacterial growth [Harutunian et al. 1996]. Several new methods are used in this paper. To reduce the bacterial growth a TiO2/Polystyrene surface treatment is used on the material. The incorporation of TiO2 nanoparticles into the polystyrene matrix increases the crystallinity and improves the barrier properties of the composites [Zhu et al. 2011]. To reduce the waste of energy, the foaming process is optimized by controlling the CO2 concentration and the pressure [Nagib et al. 2003].
2 DESIGN DECOMPOSITION AND APPLICATION OF THE AXIOMS

2.1 Axiom I

The zeroth level FR defines the overall objective of the decomposition. The objective of this project is to design a manufacturing process for a polystyrene egg carton. A linear manufacturing line for egg cartons is the physical realization for the top-level functional requirement. The first level of the decomposition follows a sequential theme. By considering the functional requirements as a series of physical transformations on the polystyrene work piece, then the decomposition is collectively exhaustive. If the transformations are independent, then the functional decomposition will also be mutually exclusive. A sequential theme suggests design parameters for a manufacturing line. This allows organizing the design parameters to avoid D-P-DP interactions. Design pictures and specified geometries can be found in the appendix.

2.1.1 CURRENT DECOMPOSITION

The current decomposition is shown in Table 1.

2.1.2 THE RELATION BETWEEN FRs AND DPS

FR1 Process Raw Polymer into Foam Sheet
DP1 Single Screw Foam Extruder

The governing equation for this function requirement is the conservation of mass. The “mass in” must equal the “mass out”. The overall decomposition for this function requirement is based on this principle. The metrics and tolerances for this can be found in Table 2.

FR1.1 Feed polystyrene pellets into extruder
DP1.1 Extruder hopper feeder

The metric for FR1.1 is the mass flow rate \( \frac{dm}{dt} \) of the polystyrene pellets into the extruder. This relates to the output flow rate of polystyrene foam according to the law of conservation of mass. Nomenclature for this equation and all others included in this paper can be found in the appendix.

\[
\text{Mass in} = \text{Mass out}
\]

\[
\frac{dm}{dt} = \pi r^2 \rho_{\text{pellets}} \frac{dx}{dt}
\]  

The density of the pellets including the void fraction is 1.033 g/cm² [Sharp, 1950].

FR1.2 Transport pellets through extruder
DP1.2 Screw pump

The metric for FR1.2 is the velocity of the polystyrene.

FR1.3 Melt pellets by heating
DP1.3 Heating element in extruder walls
The extrusion process involves a phase transition in which heating elements heat the polystyrene pellets above the melting temperature. Fourier's law expresses heat transfer for this part of the extrusion process. Fourier's Law gives the relationship between thermal conductivity and heat flow rate as shown in equation (2) [Moore, 2002]. Figure 1 implies the relationship between thermal conductivity (q), the negative local temperature gradient (k), and the temperature difference (ΔT).

\[
\frac{q}{A} = -k \frac{\Delta T}{dx}
\]  

(2)

**Figure 1. Heat transfer relationship.**

The range of melt temperatures is limited to the given range by two constraints. The viscosity of the melt must be low enough to allow it to flow smoothly through the extruder, thus the temperature must be at least 200 °C. Polystyrene begins to decompose at high temperatures, so the temperature must stay below 240°C to ensure product quality.

FR1.3.1 Create heat transfer surface area
DP1.3.1 Cylindrical surface area of extruder

The walls of the extruder provide a surface for heat transfer:

FR1.3.2 Provide heat transfer coefficient
DP1.3.2 Material and mixing property

The material in the extruder walls is thermally conductive to allow for heat transfer:

FR1.3.3 Create temperature gradient
DP1.3.3 Distance from pellets to heater

The distance provides a temperature gradient for heat transfer:

FR1.4 Create density of polystyrene foam
DP1.4 Add foaming gas agent

Figure 2 shows the interaction between density and concentration of CO₂ as a foaming agent. FR1.4 (Choice of foaming gas) was based on cost analysis. The proportion of gas to polystyrene was chosen based on mid-weight foam.

![Figure 2. Relations of CO₂ percentage, density and forming rate [Xin et al. 2008].](image)

The chosen foaming product was CO₂ because it provides good performance at a more cost efficient price [Perrot, 2009].

FR1.5 Form Foam into Sheet
DP1.5 Nozzle Unit

The final step of the extrusion process is carried out at the outlet of the extruder by mechanical tools and nozzle. The foam exits the extruder as a thin sheet. Equation (3) below relates to the children of FR1.5.

\[
\frac{dm}{dt} = \frac{dx}{dt} H D \rho_{foam}
\]  

(3)

FR1.5.1 Create desired sheet width
DP1.5.1 Width of extruder die

The die has the same width as the product.

FR1.5.2 Create desired sheet height
DP1.5.2 Lip gap of extruder output

The height of the lip is 1.3 times the desired height sheet.

FR2: Form sheet into egg carton geometry
DP2: Heating element in thermoformer

Fourier's law shows that if the flow rate is controlled, then the thermal conductivity can also be controlled and is directly related to the amount of heat needed and the duration of thermal forming. The physical component for this FR is close to the exit of the extruder as a means to reduce the energy loss in the form of wasted heat.

The temperature of the extruded polystyrene needs to be greater than that of the glass transition temperature (100°C), so that the produced sheet is amorphous and moldable.

FR2.3: Deform sheet
DP2.3: Hydraulic mold press
Table 2. Assigned Metrics.

<table>
<thead>
<tr>
<th>FR</th>
<th>Metrics</th>
<th>DP</th>
<th>Metrics</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Feed polymer pellets into extruder</td>
<td>65.5 g/s ± 2 g/s</td>
<td>Extruder hopper feed</td>
<td>Pellet density 1.033 g/mL</td>
</tr>
<tr>
<td>1.2 Transport pellets through extruder</td>
<td>160 rpm ± 2%</td>
<td>Screw pump</td>
<td>20 cm/s ± 0.2 cm/s</td>
</tr>
<tr>
<td>1.3 Create temperature gradient</td>
<td>240°C &lt; T_{min} &lt; 200°C</td>
<td>Heating element in extruder walls</td>
<td>T_{wall} - T_{inlet} = 10°C</td>
</tr>
<tr>
<td>1.4 Create desired density</td>
<td>10% mass CO₂ ± 2%</td>
<td>Inject carbon dioxide gas</td>
<td>0.05 g/cm³ ± 0.005 cm</td>
</tr>
<tr>
<td>1.5 Create desired sheet width</td>
<td>Width of sheet 131 cm ± 5 cm</td>
<td>Width of extruder die</td>
<td>131 cm ± 2 mm</td>
</tr>
<tr>
<td>1.5.2 Create desired sheet height</td>
<td>0.5 cm ± 0.03 cm</td>
<td>Lip gap</td>
<td>0.75 cm ± 0.03 cm</td>
</tr>
<tr>
<td>2.2 Maintain Temperature of Styrofoam sheets</td>
<td>T &gt; T_g ~ 100°C</td>
<td>Convoer Oven</td>
<td>T_{oven} = 120°C</td>
</tr>
</tbody>
</table>

Table 3. Design matrix.
From the law of conservation of energy, the relationship between the molding speed and the pressure can be found. Once the pressure is fixed, the speed of the mold can be calculated. Since the speed and time are fixed, then the speed of the die can be found.

\[ W = \frac{1}{2} m v^2 + P A H \]  

(4)

Table 2 contains the assigned metrics that could influence the decision-making on the design parameters and on the process variables.

### 2.1.3 COUPLING

There are several off diagonal interactions in the design matrix, as shown in Table 3:

FR1.3 Heating the polystyrene melt and DP1.2 Screw pump are coupled because the friction heating from the screw pump accounts for 90% of the heat energy in the extruder [Total Petrochemical, 2013]. Thus, the work done by the extruder should be specified first and then the heater settings adjusted to avoid iterative adjustment.

FR 1.4 Density of polystyrene foam and DP 2.3 Hydraulic mold are coupled because pressure can collapse the open cells created by the foaming process and cause the density to increase.

FR 2.2 Maintain heat and DP 2.3 Hydraulic press are linked because temperature is a function of pressure.

FR3 Surface finish and DP4 Packager: The surfaces of the cartons touching during packaging could cause deposition of the surface treatment and defects in surface finish.

### 2.2 Axiom II

Equation (6) for probability of success is shown below:

\[ I = \ln \left( \frac{p}{1-p} \right) \]  

(5)

\[ p = \frac{1}{e} \]  

(6)

For equations 5 and 6, I is the information content. In the initial design, 1 relates to each FR-DP pair and p is the corresponding probability of success.

In the initial design, there are four children for FR1.4, which produce four FR-DP pairs under FR1.4. So, the probability of success in this case equals to 0.02. Table 4 shows the initial FR-DP matrix for FR1.4.

### Table 4. Initial FR-DP matrix for FR1.4.

<table>
<thead>
<tr>
<th>Content</th>
<th>DP</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>FR1.4 Extrude foam in</td>
<td>DP1.4</td>
<td>Nozzle unit</td>
</tr>
<tr>
<td>sheet</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FR1.4.1 Control Flow Rate</td>
<td>DP1.4</td>
<td>Centrifugal pump</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FR1.4.2 Change flow shape</td>
<td>DP1.4</td>
<td>Fish tail die</td>
</tr>
<tr>
<td>from cylindrical to</td>
<td></td>
<td></td>
</tr>
<tr>
<td>flat</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FR1.4.3 Create desired</td>
<td>DP1.4</td>
<td>Lip gap</td>
</tr>
<tr>
<td>height</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FR1.4.4 Normalize flow</td>
<td>DP1.4</td>
<td>Land</td>
</tr>
<tr>
<td>regime</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The pump should be decoupled in order increase the probability to success. FR1.4 was separated and reformulated as FR1.3, FR4, and FR1.5 as shown in Table 5.

### Table 5. Adjusted FR-DP pairs.

<table>
<thead>
<tr>
<th></th>
<th>DP1</th>
<th>DP2</th>
<th>DP3</th>
<th>DP4</th>
</tr>
</thead>
<tbody>
<tr>
<td>FR1.3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>FR4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

In this condition, there are only three pairs of FR-DPs under FR1.3, so that the probability of success is improved from 2% to 5%. As for FR1.5, it would have two children with a 13.5% probability of success, and the FR-DPs becomes collectively exhausted and mutually exclusive.

### 3 PHYSICAL INTEGRATION

The physical integration matrices are shown in Table 6 and Table 7. The DPs are decomposed into each level to allow analyzing the relationships between them.

### Table 6. DP-DP interactions.

<table>
<thead>
<tr>
<th></th>
<th>DP1</th>
<th>DP2</th>
<th>DP3</th>
<th>DP4</th>
</tr>
</thead>
<tbody>
<tr>
<td>FR1.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>FR1.3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The physical integration matrix is different from the FR-DP matrix of the former section. In the FR-DP matrix, FR3 and FR4 are coupled, however here DP3 and DP4 are not coupled because finishing and packing are not physically operated at the same location. DP2.2 and DP2.3 do not interact due to the same reason.

### Table 7. Physical Integration Matrix for DP2.

<table>
<thead>
<tr>
<th></th>
<th>DP21</th>
<th>DP22</th>
<th>DP23</th>
<th>DP24</th>
</tr>
</thead>
<tbody>
<tr>
<td>FR1.1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>FR1.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

DP-DP interaction commonly happens when two physical parts are in direct contact or two processes are carried out in the same device or place. In our system, all the processes take place in different machines or different locations, so that there is no DP-DP coupling.
4 INNOVATION

A problem with current egg cartons is that surface microbial growth can lead to egg spoilage or contamination. The innovation that was added to the egg carton to address this issue was a TiO₂ nanocoating to the interior of the carton. Similar nanocoatings have been shown to have anti-microbial effects when applied to polystyrene [Loddo et al. 2012]. A spray application chamber was added to the design of the manufacturing line in order to decouple the new spraying process from the other already existing processes.

Traditional polystyrene foam egg cartons are not environment-friendly at the end of the product life cycle. A possible innovation that was explored was to use low-density polyactic acid (PLA) derived from corn as the base material for the cartons. This material has similar thermal and mechanical properties to that of polystyrene [Parker et al. 2012]. Since this material is compostable, it was felt that it would not be as beneficial to the process as the antimicrobial nanocoating.

5 CONCLUDING REMARKS

This paper shows an improvement in the design of an egg carton manufacturing process through the application of Axiomatic Design. The process is improved by new approaches that better match the functional requirements. To eliminate unnecessary coupling in the process, decomposition and mapping matrices were used to make the structure of the system more clear, so that the processes could be rearranged. The improved process would reduce production time and energy waste. In addition, the eggs will have better protection against bacteria via the newly designed egg carton surface coating. Future works will include improving the material’s performance, especially what concerns to the environmental aspects, and recycled polystyrene and biodegradable materials will be considered.

6 ACKNOWLEDGMENT

This project was part of “Axiomatic Design of Manufacturing Processes”, a WPI graduate course. A special thanks is due to Professor Christopher Brown, as well as to Axiomatic Design Solutions, Inc. for the use of Acdario DFSS software.

7 REFERENCES


Axiomatic Design of an Improved Egg Carton Manufacturing Process
The Eighth International Conference on Axiomatic Design
Campus de Caparica - September 24-26, 209

Copyright ©


8 APPENDICES

<table>
<thead>
<tr>
<th>DP1: Single Screw Foam Extruder</th>
<th>DP2</th>
<th>DP3</th>
<th>DP4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DP2: Thermoforming Press</th>
<th>DP1</th>
<th>DP2</th>
<th>DP3</th>
<th>DP4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DP3: Finishing Processes</th>
<th>DP1</th>
<th>DP2</th>
<th>DP3</th>
<th>DP4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DP4: Package</th>
<th>DP1</th>
<th>DP2</th>
<th>DP3</th>
<th>DP4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

DP1.5.1: Fish tail die
DP1.5.2: Lip gap
DP1.5.3: Land

<table>
<thead>
<tr>
<th>DP2.1: Mold feeder tray</th>
<th>DP2.2</th>
<th>DP2.3</th>
<th>DP2.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DP2.1: Oven conveyor</th>
<th>DP1</th>
<th>DP2</th>
<th>DP3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DP2.3: Hydraulic mold press</th>
<th>DP1</th>
<th>DP2</th>
<th>DP3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DP2.4: Carton extractor</th>
<th>DP1</th>
<th>DP2</th>
<th>DP3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DP3.1: Spray coating applicator</th>
<th>DP3.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DP3.2: Ink jet printer</th>
<th>DP3.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DP4.1: Vertical punch press</th>
<th>DP4.2</th>
<th>DP4.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DP4.2: Punch stacking machine</th>
<th>DP4.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DP4.3: Shrink wrapping machine</th>
<th>DP4.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Nomenclature

H - depth of foam sheet (cm)
D - width of foam sheet (cm)
\( \rho_{\text{Foam}} \) - density of foam sheet (g/cm³)
\( \frac{dx}{dt} \) - rate of foam output (cm/s)
\( \frac{dm}{dt} \) - mass flow rate of sheet (g/s)
\( \frac{\Delta}{\Delta x} \) - heat flow rate per unit area
\( k \) - thermal conductivity
\( \frac{dT}{dt} \) - temperature (°C) gradient in the direction of heat flow, x
\( W \) - total energy supplied by the electrical system of the molding machine (J)
H - vertical movement distance of the die (cm)
\( v \) - speed of the upper mold (cm/s)
\( m \) - mass of the upper mold (g)
P - pressure applied to the product (Pa)
\( A \) - area (cm²)