Abstract

Computing the information content of coupled designs is seldom discussed in the literature, probably because the Axiomatic Design (AD) practitioners know that coupled design solutions should be avoided. On the other hand, Suh's theorem 7 states, “the information contents of coupled and decoupled designs depend on the sequence by which the DPs are changed to satisfy the given set of FRs”. From this theorem, one could be tempted to conclude that the information contents of coupled designs cannot be computed, because they have not a “right” sequence for changing the values of the DPs in order to satisfy the given FRs. This misunderstanding could then be used to stress that AD is not useful as a decision-making approach for coupled designs. Yet, coupled designs do exist, they are many times unavoidable and their information contents can be computed, although this is often hard to perform. This paper presents the computation of the information content for the simple case of a 2-FR, 2-DP coupled design and illustrates how this topic is related to Suh’s theorem 8 on independence and tolerance.

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1. Introduction

The large majority of the design methodologies of the 20th century follow the costly and time-consuming cycle “design-model-test-redesign-model-test” or, even worst, “design-build-test-redesign-build-test”, when physical prototype testing is required due to uncertainty. This is, for example, the case of the methodologies proposed by Pahl and Beitz [1] or by Hubka [2]. This drawback, which is common to the traditional heuristic methodologies, turned out to be crucial some years after the World War II, when an overwhelming demand of new high-quality products with a short time-to-market came into play. Definitely, something new and methodologically different should happen to allow engineers to break the above said development cycle, so that they could consistently “do it right at the first time”, timely and at an affordable cost.

In the end of the 1970s, Nam Pyo Suh introduced a new engineering design theory that was made known to the public in 1990, under the name of Axiomatic Design (AD), through a seminal book on the subject [3]. Suh’s motivation was to provide scholars and designers with a theoretical foundation for design that follows the pattern and the criteria of modern science, as to stimulate substantial improvements in teaching and in practicing design.

The wide scope of AD makes it valuable in any functionally-driven design context, especially in conceptual design; and its breath is so large that it proved its usefulness outside the traditional engineering fields, such as in the planning of intricate surgery sessions [4], in the management of healthcare systems [5], or in approaches to raise venture capital [6].

In AD, design is regarded as an intellectual endeavor that could be described as decision-making process, which success depends on the accurate knowledge about the functional goals and constraints, as well as on the mastering of the engineering sciences and the technologies related to the likely alternative design solutions. The AD fundamental decision criteria are stated in the form of two axioms: the independence axiom and the information axiom [3].
The independence axiom states that the functional requirements (FRs) of any good design solution should be fulfilled in an independent manner, while the information axiom states that best of the alternative solutions is the one with the minimum information content.

Two kinds of design solutions fulfill the independence axiom: uncoupled designs, in which the values of the design parameters (DPs) can be adjusted in an arbitrary order, and decoupled designs in which the values of the DPs can be set in a certain order, so that setting the value of each DP only impacts one FR. A third kind of designs exists: the coupled designs that breach the independence axiom and therefore should be avoided.

Tackling new designs should begin by trying to fulfill the independence axiom. This allows identifying the good and the poor alternative solutions, and the next step is to select the best solution, for which the information content is minimum, as per the information axiom.

But what if all the alternatives are poor? The AD’s traditional approach is to look for more alternative solutions until at least an uncoupled or decoupled solution is found. In this paper, we argue that there are many cases where one cannot find any good solution, at least in a realistic term, case of which one could have to compare two or more coupled designs to make a decision.

Thus, this paper presents a method for the computation of the information contents for the simple case of a 2-FR, 2-DP coupled design and illustrates how this topic is related to Suh’s theorem 8 on independence and tolerance.

2. Trying to decouple some typical coupled designs

Coupled designs occur very often. In “design for cost”, for example, cost can be taken as a requirement. In this case, adjusting the value of any design parameter would impact the cost of the product, which will become a coupled design.

Decoupling the coupled designs might be tried through the two following approaches. The first approach is to take one or more design specifications as input constraints, and not as requirements. This might reduce the number of couplings, because the constraints do not integrate the design equation. Yet, doing it right at the first time means that the design specifications should be taken pro-actively as requirements, and not as constraints, since checking any solution against the constraints could only be made after a set of DPs is previously selected, that is, after a functionally viable design solution is found. The second approach is to use the “plant some trees” metaphor that could be briefly explained through a simple example.

Let us suppose that one wants to design a small electric power station that should burn some kind of fossil fuel. Therefore, let us consider that we have a functional requirement, the nominal power of the power station, \( P \), and an eco-requirement, the rate of \( CO_2 \) emissions, \( E \). A potential physical solution with two design parameters, the power rating of the generators that we are going to use, \( g \), and the size of the power station in terms of number of generators, \( n \), could be explored. In this case, we have a coupled design solution depicted by the design equation

\[
\begin{bmatrix}
 P \\
 E 
\end{bmatrix} = 
\begin{bmatrix}
 x & x & g \\
 x & x & n 
\end{bmatrix},
\]

(1)

where \( x \) denotes the non-zero elements of the design matrix.

Equation (1) shows that we could achieve the nominal electric power of the power station by adjusting both the design parameters \( g \) and \( n \). Doing so, we cannot use either \( g \) and \( n \) to achieve the targeted rate of the \( CO_2 \) emissions, because this would disturb the previously attained value for \( P \).

This is the point where the metaphor comes into play: we “plant some trees”, in number of \( t \), in order to counterbalance the harmful effects of the \( CO_2 \) emissions with the help of the trees’ photosynthetic action. Adding the trees does not impact the produced electric power and the design equation becomes

\[
\begin{bmatrix}
 P \\
 E 
\end{bmatrix} = 
\begin{bmatrix}
 x & x & 0 \\
 x & x & n 
\end{bmatrix},
\]

(2)

which right trapezoidal design matrix denotes a redundant decoupled design, as shown elsewhere [7]. Nevertheless, in other designs it is often hard, if not impossible, to find out the right “trees” that one has to “plant”.

Incidentally, things are not always so simple and most of the eco-designs are coupled. This is easy to realize through the case study presented by Shin et al. [8] that found a design solution for a flashlight by making some design decisions through the appraisal of the flashlight’s eco-friendliness. They started from a 12-FR, 12-DP deeply coupled design solution that proved impossible to decouple. Next, they used the Life Cycle Assessment (LCA) technique to find out three extra eco-functional requirements (eFRs) that were used to obtain the so-called augmented design matrix with 15-FR, 12-DP. This matrix also corresponds to a coupled solution according to Suh’s theorem 1, which states, “When the number of DPs is less than the number of FRs, either a coupled design results or the FRs cannot be satisfied.” [3, pp. 56-57].

Shin et al. [8] eventually succeeded to find a lesser-coupled design and could compare, under an ecological standpoint, the use of different materials to build the flashlight. Yet, LCA does not match the AD scientific goal, given that it uses benchmarks to assess the relative impact of eco-issues. On top of the subjectivity of benchmarking, LCA involves applying weighting factors in the appraisal of the joint impact of multiple eco-issues. This contravenes Suh’s theorem 16 that states, “All information contents that are relevant to the design task are equally important regardless of their physical origin, and no weighting factor should be applied to them.” [3, p. 321].

3. Why computing the information of coupled designs?

Coupled design solutions do exist and often we cannot avoid them. Nevertheless, other than the topologic structure of their design matrices, coupled designs are perhaps the less learned topic of AD.

The supremacy of AD becomes clear in conceptual design, when the first decisions are made with the help of the
independence axiom. Further decisions are supported by the information axiom as well. On the other hand, the engineering sciences, such as continuum mechanics, sometimes together with multiple-criteria analysis, are used to do detailed design. Such an approach means that designers have to deal with coupled solutions from time to time.

Therefore, there is no reason to neglect the computation of the information contents of coupled designs, no matter how difficult this could look like. In addition, it is worth remarking that among the potential newcomers to AD, a meaningful number of practitioners that are acquainted with the heuristic design methodologies hardly accept the axiomatic approach [9]; and we wonder if AD’s usual boldness about coupled designs is one of the causes of their reluctance.

Nonetheless, we argue that many detailed design solutions have more than one FR, and that many of them are coupled.

For example, let us consider the bare choice of a standard steel roller chain drive by using a typical brand catalogue. The center distance and the number of the teeth of the two sprockets of the drive were previously fixed, so that the FRs are the power rating of the drive, \( P \), and its life expectancy at full power, \( L \).

The design parameters are the pitch of the chain, \( p \), and rotational speed of the smaller sprocket, \( n \). The power rating depends on the difference between the static tensile strength of the chain and the load induced in the chain by the centripetal forces. This means that \( P \) depends on \( p \) and on \( n \). On the other hand, the life expectancy of a properly operated drive is governed by the wear of the pins and bushings due to the cyclical rocking of the chain links that occurs when they engage or disengage the sprockets. As such, the life expectancy depends on the actual tensile load applied to the chain, and on the number of times each chain link engages and disengages the sprockets. Therefore, \( L \) also depends on \( p \) and on \( n \), and the design matrix of the drive is described by Eq. (3) and corresponds to a coupled design

\[
\begin{bmatrix}
P \\
L \\
\end{bmatrix} = \begin{bmatrix} x & x \\ x & x \end{bmatrix} \begin{bmatrix} p \\ n \end{bmatrix}.
\]

Should we try to develop a new non-standard chain drive and we would find much more DPs for the very same FRs, and nobody could convince anybody to avoid using chain drives just because they are coupled designs.

Our motivation for better studying the computation of the information contents of coupled designs is therefore twofold: 1) To contribute to the advancement of AD as a scientific theory of design; 2) To foster the attractiveness of AD.

4. The information contents of coupled designs - \( \delta \text{FR} \) plots

Shin et al. [10] briefly showed that the information contents of 2-FR, 2-DP coupled designs could be computed, either using graphical or integration methods, and noticed that the graphical method is not suitable for other designs. Suh [11] and Park [12] used joint probability to explain in detail how to compute the information contents for 2-FR, 2-DP decoupled designs, both graphically and by integration. Park noticed that these methods could be modified to consider coupled designs [12]. Yet, he did not give any detail about the required modifications due to their complexity and maybe because his opinion is that coupled designs are not considered in general design [12, pp. 44-45]. Later, he gave a hint for his option stating, “In detailed design, there is a design variable vector which consists of many design variables. An FR of axiomatic design is equivalent to the objective function of detailed design while a DP of axiomatic design is equivalent to the design variable vector of detailed design. The detailed design process is a one FR-one DP problem from the axiomatic design viewpoint. Therefore, the independence axiom is automatically satisfied and the detailed design process is similar to the process of applying the information axiom.” [9].

Clearly, the citation describes a 1-FR redundant design that could be determined through the typical analytic techniques of the engineering sciences.

As for the information contents of multiple FR designs, Suh’s theorem 12 states, “The sum of information for a set of events is also information, provided the proper conditional probabilities are used when the events are not statistically independent.” [3, p. 153], and this assertion should hold for all kinds of designs, uncoupled, decoupled and coupled.

The trouble arises when one comes to coupled designs, because their information content is governed by conditional probability and the result that is attained is path-dependent. On the contrary of the decoupled designs, however, there is not a definite path to tune the FRs of the coupled designs.

Shin et al. [10] surpassed this annoyance by modifying the versions of the procedures that were applied by Suh [11] and Park [12] for 2-FR, 2-DP decoupled designs.

The procedures of Suh [11] and Park [12] assume uniform probability distributions, and might be in need of Suh’s theorem 17 that states, “Design can proceed even in the absence of complete information only in the case of uncoupled design if the missing information is related to the off-diagonal elements.” [11, p. 52]. In other words, the theorem says that when the off-diagonal elements are unknown, the computation of the information contents of uncoupled designs can proceed as if they were uncoupled.

It seems that Shin et al. [10] tacitly used this theorem since there is not any reason to preclude the coupled designs, a condition that Suh did not consider, maybe due to the harmful effects that arise from infringing the independence axiom, as shown by Hilario L. Oh [11, pp. 185-187].

The coupled design of Shin et al. [10] has the following design equation, with \( A_{ij} \geq 0 \) and \( \Delta DP \), statistically independent

\[
\begin{bmatrix}
\delta \text{FR}_1 \\
\delta \text{FR}_2 \\
\delta \text{DP}_1 \\
\delta \text{DP}_2 \\
\end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\
A_{21} & A_{22} \end{bmatrix} \begin{bmatrix}
\delta \text{FR}_1 \\
\delta \text{FR}_2 \\
\delta \text{DP}_1 \\
\delta \text{DP}_2 \\
\end{bmatrix}.
\]

The design of this equation was studied in the neighborhood of the point \( (\text{FR}_1', \text{FR}_2') \) that is defined by

\[
\begin{align*}
\text{FR}_1' &- \Delta \text{FR}_1' \leq \text{FR}_1' \leq \text{FR}_1' + \Delta \text{FR}_1' \\
\text{FR}_2' &- \Delta \text{FR}_2' \leq \text{FR}_2' \leq \text{FR}_2' + \Delta \text{FR}_2'
\end{align*}
\]
The graphical procedure of Park [12] in the FR space for the case of 2-FR, 2-DP decoupled designs was adapted to coupled designs with uniform probability distributions, as shown in Fig. 1. The common range is the grey square resulting from the intersection of the square design range with the system range, which is the white parallelogram shaped by the two pairs of straight contour lines related to $FR_1$ and $FR_2$. The problem is seen as linear since $\Delta FR_1$ and $\Delta FR_2$ are small.

In Fig. 1, the random variation of $FR_i$ in the aforesaid neighborhood is denoted by $\delta FR_i$ and there is no bias. The information content of the design is given by

$$ I = \log_2 \frac{\text{Area of the System Range}}{\text{Area of the Common Range}}. \quad (5) $$

Fig. 2 represents the same design because the system range is the same of Fig. 1; but now the area of the design range is larger, so that the grey hexagon depicts the area of the common range. Notice that the actual value of the information content of Fig. 2 is not equal to the one that is given by Eq. (5) because we cannot consider the design as decoupled or uncoupled and we should have used conditional probability.

In an unbiased condition, the design can be regarded as uncoupled if the design range is a rectangle that does not overdo the points $A$, $B$, $C$ and $D$ that belong to the system range, as shown in Fig. 3.

This is the limit condition that matches Suh’s theorem 8, which states that a design can be considered as uncoupled when the specified tolerance of $FR_i$, which is denoted by $T_i$, is such that

$$ T_i \geq \sum_{j \neq i} \left( \frac{\partial FR_i}{\partial DP_j} \right) \Delta DP_j, \quad (6) $$

so that the non-diagonal elements of the design matrix can be neglected from design consideration [3, p. 122]. In the current case, one has

$$ T_1 \geq A_{12} \Delta DP_2, \quad T_2 \geq A_{21} \Delta DP_1. \quad (7) $$
Incidentally, this means that theorem 17 related to the importance of the off-diagonal elements of decoupled designs also holds for coupled designs. Thus, Eq. (5) holds for Fig. 3.

The coordinates of points $A$, $B$, $C$ and $D$ of the system range are given by

$$A = \{ 0, +y \}, \quad B = \{ +x, 0 \},$$

$$C = \{ 0, -y \}, \quad D = \{ -x, 0 \},$$

where

$$x = \frac{A_{11} A_{22} - A_{12} A_{21}}{A_{22}},$$

$$y = \frac{A_{12} A_{21} - A_{11} A_{22}}{A_{22}}.$$

The corresponding integration method is not studied here for a matter of simplicity; but, as noticed by Park [12, p. 38], the method is required to tackle multiple-FR, multiple-DP designs, or when the probability density distributions are not uniform.

### 5. The information contents of coupled designs - $\delta$DP plots

Section 4 shows the graphical method using $\delta$FR plots for 2-FR, 2-DP unbiased coupled designs with uniform probability density distributions.

The plots of Figs. 1, 2 and 3 might be converted into $\delta$DP$_1$, $\delta$DP$_2$ plots. These plots are useful to deal with the responses of systems (which are seen as the systems’ FRs) vs. the random variation of $\delta$DP, and $\delta$DP$_2$.

Fig. 4 depicts the $\delta$DP$_1$, $\delta$DP$_2$ plot of the 2-FR, 2-DP design of Fig. 3, which is described by Eq. (4). The $\delta$DP range was chosen so that the design could be regarded as uncoupled.

In Fig. 4, this limit condition corresponds to the rectangle passing through the points $A$, $B$, $C$ and $D$, which coordinates depend on the FR tolerances, as per Eq. (7). Therefore, the $\delta$DP range must be such that

$$|\delta$DP$_1| \leq \frac{\Delta FR_1}{A_{21}}, \quad |\delta$DP$_2| \leq \frac{\Delta FR_2}{A_{22}}.$$

Because the design can be seen as uncoupled, its information content can be computed through Eq. (5), where the system range is the parallelogram formed by the contour lines and the common range is the grey octagon of Fig. 4.

The coordinates of points $A$, $B$, $C$ and $D$ that belong to the system range are

$$A \equiv \{ 0, +y \}, \quad B \equiv \{ +x, 0 \},$$

$$C \equiv \{ 0, -y \}, \quad D \equiv \{ -x, 0 \},$$

where

$$x = \frac{\Delta FR_1}{A_{11}},$$

$$y = \frac{\Delta FR_2}{A_{22}}.$$
The new design of Fig. 5 was obtained by adding the functional requirement $FR_j$ to the design of Fig. 4. It is depicted by a $\delta DP$ plot that differs from the one of Fig. 4 in that one can identify the new pair of contour lines that correspond to $\Delta FR_j$.

In Fig. 5, the largest $\delta DP$ range that allows considering the design as uncoupled is the dark grey rectangle passing through points A, B, C and D. The vertical and horizontal straight lines that pass through points E, F, G and H, where the new contour lines intersect the contour lines that came from Fig. 4, delimit this dark grey rectangle. The new system range is represented by the light grey hexagon of Fig. 5.

According to Suh’s theorem 8, this means that

$$T_1 \geq \frac{\partial FR_i}{\partial DP_1} \Delta DP_2 = A_{i2} \Delta DP_2$$
$$T_2 \geq \frac{\partial FR_i}{\partial DP_2} \Delta DP_1 = A_{i2} \Delta DP_1$$
$$T_3 \geq \frac{\partial FR_i}{\partial DP_1} \Delta DP_1 + \frac{\partial FR_i}{\partial DP_2} \Delta DP_2 = A_{i1} \Delta DP_1 + A_{i2} \Delta DP_2$$

Under these conditions, the design of Fig. 5 can be considered uncoupled and its information content can be computed through Eq. (5).

The coordinates of the points A, B, C and D of Fig. 5 are

$$A \equiv (0, +y), \quad B \equiv (+x, 0),$$
$$C \equiv (0, -y), \quad D \equiv (-x, 0),$$

where

$$x = \frac{A_{i2} \Delta FR_2 - A_{i1} \Delta FR_1}{A_{i2} A_{i1} - A_{i1} A_{i2}}, \quad y = \frac{A_{i1} \Delta FR_1 - A_{i2} \Delta FR_1 + \Delta FR_1}{A_{i2} A_{i1} - A_{i1} A_{i2}}$$

Comparing Figs 4 and 5, which were drafted at the same scale, one can see that the information content of the 3-FR design of Fig. 5 is larger than the one of the 2-FR design of Fig. 4. In fact, adding a third FR to a 2-FR, 2-DP design typically reduces the probability of success.

It is worth noticing that converting $\delta DP_1$, $\delta DP_2$ plots to $\delta FR$ plots is not possible when the designs have more than two FRs.

6. Conclusion

According to Axiomatic Design, coupled design solutions result from poor design decisions and should be avoided. One cannot get away from coupled solutions every time. Therefore, we argue that they deserve our attention, although coupled designs usually have poor robustness and poor adaptability, which is meant as the easiness for changing the values of their FRs to match changes in the operating conditions that might occur after the designs are fielded. On this matter, the best condition is the prospect of considering coupled designs as uncoupled, and this was the main motivation of this paper.

Furthermore, coupled designs have not a “right” order for changing the values of the DPs in order to satisfy the FRs. Thus, computing the information content of coupled designs might challenge Suh’s theorem 7 that states, “The information content of coupled and decoupled designs depend on the sequence for changing DPs to satisfy the given set of FRs”.

Thus, there is not an accurate value for the information content of coupled designs that cannot be seen as decoupled or uncoupled, since the probability of fulfilling FR depends on the probability of fulfilling $FR_1$ and vice-versa. Yet, Suh’s theorems 8 and 17 allow surpassing the problem by establishing the conditions in which they can be assumed as uncoupled.

The paper shows how to use Suh’s theorem 8 to establish the maximum size of the design range of 2-FR, 2-DP unbiased coupled designs with uniform probability density distributions, so that they can be regarded as uncoupled. It also details a graphical method based on $\delta FR$ plots to compute the information content of unbiased 2-FR, 2-DP coupled designs, and a graphical method to compute the information content of such designs based on $\delta DP$ plots, using Suh’s theorem 8 to establish the maximum size of the DP range that ensures they can be regarded as uncoupled. At last, the paper introduces a new graphical method to compute the information content of multiple-FR, 2-DP unbiased coupled designs with uniform probability density distributions.

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