Reliability-oriented complexity analysis of manufacturing systems based on fuzzy axiomatic domain mapping

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Abstract

Decreasing the complexity is the effective way to improve the reliability of manufacturing systems. However, the research on the reliability oriented complexity analysis for manufacturing system is rare. A reliable manufacturing system is a prerequisite to ensure the well-designed products be manufactured faultlessly. In this context, this paper proposes an axiomatic complexity analysis model of manufacturing system based on the fuzzy axiomatic domain mapping. Firstly, in terms of the uncertainty of domain mapping information, the fuzzy evaluation matrix for complexity which integrated the weights of experts is constructed based on triangular fuzzy numbers. Secondly, considering the influence of the design parameters (DPs) on the functional requirements (FRs), a quantitative computation approach to manufacturing system complexity is achieved by means of fuzzy evaluation matrix. Finally, a case study is presented to illustrate the validity of the proposed method.

Keywords: Axiomatic design; manufacturing system; reliability analysis; complexity modeling; fuzzy theory;

1. Introduction

Complexity continues to be one of the biggest challenges faced by the modern manufacturing today. A reliable manufacturing system is a prerequisite to ensure that well-designed products can be manufactured faultlessly [1]. With the dynamic development in customer demands and design techniques, the complexity of manufacturing systems is also increased accordingly in both physical and functional domains [2]. Decreasing complexity is an effective way to improve reliability of manufacturing systems [3,4]. Therefore, the complexity analysis of manufacturing systems has been extensively studied by many scholars.

Researchers have studied the complexity of manufacturing systems from different perspectives. Taking into account that the layout of manufacturing systems determines its structural complexity, Elmaraghy et al. [5] proposed a model which is used to evaluate the structural complexity of manufacturing systems layout in the physical domain. Samy et al. [6] developed a new system granularity complexity index, which could sum up and normalize the complexity resulting from the system layout complexity and the equipment structural complexity. Bozarth et al. [7] analyzed the impact of supply chain complexity on the performance of manufacturing systems. Hu et al. [8] defined complexity as an entropy function and proposed a unified measure of complexity to assist in designing multi-stage assembly systems with robust performances. Shannon’s information theory/entropy approach provides a new way for complexity modeling and commonly used as the underlying basis for quantifying the complexity [9]. From the view of information theory, many studies have analyzed the complexity of manufacturing systems [10,11]. With all the documents mentioned above, not only is the meaning of complexity of manufacturing systems defined from multiple viewpoints, but also the complexity of manufacturing systems is described qualitatively and quantitatively by analyzing the relationships among equipment, components, and manufacturing systems. These studies make the research on the complexity analysis of manufacturing systems become scientific and systematic gradually.

Suh [12] presented the Axiomatic design (AD) theory and defined complexity as the uncertainty in achieving the functional requirements that need to be met, which provides a new theoretical basis for complexity analysis of manufacturing systems from the viewpoint of quality design.
However, in real case problems, the relations between FRs and DPs may be unknown or uncertain, and the expert’s descriptions may be fuzzy, which will lead to the uncertainty of the relations between FRs and DPs are difficult to represent rightly. Moreover, conventional qualitative evaluation of the design parameters constrains the designer to understand the complexity of manufacturing systems accurately. And then the reliability-oriented complexity analysis of manufacturing systems is prevented.

As can be seen from the above analysis, complexity breeds couplings and defects, and coupling reduces reliability. Complexity analysis of manufacturing systems can provide feedback and promote the reliability evolution of the manufacturing system. Driven by these requirements, a novel approach to reliability-oriented complexity analysis of manufacturing systems based on fuzzy axiomatic domain mapping is proposed in this paper.

2. The foundations of complexity modeling for manufacturing systems

2.1. Fundamentals of axiomatic design

The axiomatic design (AD) theory was proposed by Suh [13], which is typically used to guide designers to use all existing design tools to get a successful new design or to diagnose and improve an existing design. AD theory consists of four domains and two axioms. Four domains respectively refer to Customer Domain defined by Customer Attributes (CAs), Functional Domain for defining Functional Requirements (FRs) and constraints, Physical Domain for representing Design Parameters (DPs), and Process Domain for characterization of Process Variables (PVs).

Axioms are widely accepted principles which are fundamental to assure the mapping quality of the four domains. They are stated as follows:

\**Axiom 1. Independence axiom**

Maintain the independence of the FRs.

\**Axiom 2. Information axiom**

Minimize the information content of the design scheme.

Independence axiom, the first axiom of the AD principles, is about maintaining the independence between FRs. The mapping relationship between FRs and DPs can be expressed as:

\[ \{FRs\} = [A]\{DPs\}, \quad A = [A_{ij}]_{m \times n} \quad (1) \]

\[ A = \begin{bmatrix}
A_{11} & A_{12} & \cdots & A_{1n} \\
A_{21} & A_{22} & \cdots & A_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
A_{m1} & A_{m2} & \cdots & A_{mn}
\end{bmatrix} \quad (2) \]

\[ A_{ij} = \frac{\partial FR_i}{\partial DP_j} \quad (3) \]

where \( \{FRs\} \) is the collection of FR vectors, \( \{DPs\} \) are the collection of DP vectors, and \([A]\) is the design matrix.

Information axiom, the second axiom of the AD principles, is about minimizing the information content of the design. Since the information content is determined by probability. The axiom also shows that the design with the highest probability of success is the best [14]. The positive significance of the information axiom lies in that it has provided a criterion for evaluating the quality of design. If the probability of success for a given FR is \( p_i \), the information content can be denoted as:

\[ I_i = \log_2 \frac{1}{p_i} \quad (4) \]

If there is more than one FR, the information content can be calculated as:

\[ I_{system} = \sum_{i=1}^{n} \log_2 \frac{1}{p_i} \quad (5) \]

2.2. Manufacturing systems complexity

Manufacturing system complexity is usually divided into dynamic complexity and static complexity. The dynamic complexity is mainly related to real-time operation and material flow pattern, which emphasizes the complexity of the operating state of manufacturing systems [15]. However, in the design stage of manufacturing systems, its complexity is mainly characterized by the static complexity that determined by system structure and components. The focus of this paper is on the mapping relationship between the functional domain and the physical domain in the design process, namely the static complexity.

The complexity of manufacturing systems increases with the increase of the coupling degree and uncertainty. A higher complex system often requires a larger amount of information to describe the system state. That is, an increasing complexity of a system, through increased coupling, variety, and uncertainty, will increase its information content [14]. Therefore, the complexity of manufacturing systems could be measured by the information content.

2.3. Fuzzy axiomatic domain mapping theory

The relation matrix which is used to characterize the correlation between functional requirements and design parameters consists of “0” and “1”. Those symbolize whether there are relations between FRs and DPs. However, in dealing with practical problems, there is always a potential or very small or indirect relationship or even an unknown relationship between an FR and a DP. And the descriptions of these relations are often fuzzy, such as medium, higher, lower, very high and so on. In order to clearly express and make full use of these fuzzy information, fuzzy axiomatic domain mapping theory, a better solution to the problem of fuzzy information in evaluation and decision-making is proposed, which refers to adopt the triangular fuzzy number method into the “Zig”
mapping between adjacent domains in AD, and quantify the qualitative fuzzy information. As shown in Fig.1.

As shown in Fig.2, seven linguistic terms, namely none, very low, low, medium, high, very high and excellent, are defined in a range from 0 to 1. Based on the language scale, the fuzzy relation between FRs and DPs can be defined on the interval ranging from 0 to 1.

3. Fuzzy complexity modeling method of manufacturing systems

3.1. Modeling framework

In order to analyze the static complexity of manufacturing systems in a module by zigzagging between domains, and determine the design matrix, as in Eq.6. Then, the evaluation of these relations can be implemented with the aid of experts, and the linguistic evaluation matrix $\tilde{D}_i$ can be gotten, as in Eq.7.

$$\tilde{D}_i = [\tilde{u}_{ij}] = 
\begin{bmatrix}
\tilde{u}_{11} & \tilde{u}_{12} & \cdots & \tilde{u}_{1n} \\
\tilde{u}_{21} & \tilde{u}_{22} & \cdots & \tilde{u}_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{u}_{m1} & \tilde{u}_{m2} & \cdots & \tilde{u}_{mn}
\end{bmatrix}
$$

where $\tilde{u}_{ij}$ is the linguistic evaluation value of the expert $e$ to the $j$th FR for the $i$th DP. Based on the Eq.7, transform the linguistic valuation matrix into the fuzzy evaluation matrix $\tilde{A}_e$ expressed by a triangular fuzzy number $\tilde{a}_{ij} = (\tilde{a}_{ij}, a_{ijl}, a_{ijh})$, as in Eq.8.

$$\tilde{a}_{ij} = [\tilde{a}_{ij}] = 
\begin{bmatrix}
\tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\
\tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & \tilde{a}_{mn}
\end{bmatrix}
$$

3.2. Specific algorithm in modeling process

The overview and steps of the fuzzy complexity modeling method are introduced in Section 3.1. and each step will be explained in detail below.

Step 1. Construction of fuzzy evaluation matrix

Firstly, define FRs in the functional domain and DPs in the physical domain in a module by zigzagging between domains, and determine the design matrix, as in Eq.6. Then, the evaluation of these relations can be implemented with the aid of experts, and the linguistic evaluation matrix $\tilde{D}_i$ can be gotten, as in Eq.7.
Step 2. Construction of fuzzy comprehensive evaluation matrix

The total number of experts is $M$. Assuming that the prior weight of each expert is $\omega_{ki} = \frac{1}{M}$. Then, the consistency of an evaluation matrix with the remaining $M-1$ evaluation matrices can be expressed as $\frac{1}{M-1} \sum_{k=1, k \neq i}^{M} S(A_i, A_k)$. Based on the calculation method of similarity degree proposed by Chen and Liu [16],

The final correction weights of each expert can be obtained by the convex combination of the prior weight and the posterior weight, as in Eq.10.

$$\omega_{ki}^e = \gamma \omega_{ki} \omega_i = (1-\gamma) \omega_{ki}^e$$

where $0 \leq \gamma \leq 1$, which reflects the preference of the weights of the experts.

Finally, the comprehensive evaluation matrix can be denoted as:

$$A = \omega_1^e \otimes \hat{A}_1 \otimes \omega_2^e \otimes \hat{A}_2 \otimes \cdots \otimes \omega_M^e \otimes \hat{A}_M$$

Step 3. Computation of fuzzy coupling degree

Based on the fuzzy comprehensive matrix that was obtained by step 2, the fuzzy coupling degree can be calculated by Eq.12 and Eq.13 proposed by Cebi [17].

$$\hat{C} = \frac{\sum \sum a_{ij}}{n^2}$$

If $\hat{C} = 1$ or $\hat{C} > \delta$, the design is coupled. If $\hat{C} = 0$ or $\hat{C} \leq \delta$ and:

$$\tilde{C} = \frac{\sum \sum a_{ij}}{n^2}$$

where $\hat{C}$ and $\tilde{C}$ indicate the index of coupling degree, $\delta$ is the value which shows the tolerable relation defined by experts. When the value of $\hat{C}$ is 0, design is uncoupled. Otherwise, design is decoupled. Therefore, if the design is coupled, the coupling degree is $c = (\hat{C} + \tilde{C})$, otherwise, $c = \tilde{C}$.

Therefore, the coupling degree with removing fuzziness of a set of FRs ($k$) can be denoted as:

$$E(c_k) = \left(\frac{a_1 + 2a_m + a_h}{4}\right)$$

where $a_1, a_m, a_h$ are the fuzzy numbers in $c$.

Step 4. Computation of fuzzy information content

According to the above analysis, two factors will affect the information content: the uncertainty of the component functions and the coupling degree of design. The coupling degree of design can be calculated by Eq.14, and the functional uncertainty of component is represented by inherent reliability. Assuming that the function of each component is a series relation in a set of FRs, that is, any component fails, the set of FRs cannot be satisfied. The inherent reliability of component ($i$) is $R_i$.

Therefore, based on the Eq.4, the fuzzy information content can be calculated below:

$$I_k = -(1 + E(c_k)) \log_2 \left(\prod_{i=1}^{n} R_i \right)$$ (15)

In the design process of manufacturing systems, the hierarchy levels from the top to the bottom in a functional domain and physical domain reflects the evolution of the design process from early stages to more detailed stages. Therefore, the complexity of the top layer can be expressed by the fuzzy information content of the bottom layer. Assuming that each function module is independent of each other, then the system complexity can be denoted as follows:

$$I_{system} = -(1 + \sum_{i=1}^{K} E(c_i)) \log_2 \left(\prod_{k=1}^{n} \prod_{i=1}^{l} R_i \right)$$ (16)

where $K$ is the total number of functional modules.

4. Case study

In this part, reliability-oriented complexity analysis of a furniture manufacturing system is discussed. In the design stage, FRs required by the production requirements are defined in the functional domain, and the equipment and components which are needed to perform the production activities will be defined in the physical domain. The main design requirement for the new manufacturing system is to maximize the return on investment. To achieve these goals, three requirements are defined by management:

- Increase production of 150% to 170%.
- Reduce production cost of one piece product.
- Improve the utilization rate of machines.

The partial Functional Requirements (FRs) and Design Parameters (DPs) are collected as the analysis object, and the results are shown as follows.

<table>
<thead>
<tr>
<th>Level</th>
<th>FRs</th>
<th>Description</th>
<th>DPs</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 1</td>
<td>FR1</td>
<td>Manufacture 1000 finished goods/day according to specs.</td>
<td>DP1</td>
<td>A manufacturing system capable of producing 1000 required goods.</td>
</tr>
<tr>
<td>Level 2</td>
<td>FR11</td>
<td>Make x writing desks/day according to specs.</td>
<td>DP11</td>
<td>Machines (cutting, edge banding, etc.) to make writing desk.</td>
</tr>
<tr>
<td>Level 2</td>
<td>FR12</td>
<td>Make y cupboards/day according to specs.</td>
<td>DP12</td>
<td>Machines (cutting, edge banding, etc.) to make cupboard.</td>
</tr>
<tr>
<td>Level 3</td>
<td>FR11</td>
<td>Make x gables with drilling pattern GR1022 according to specs.</td>
<td>DP11</td>
<td>Machines for processing gables with drill head.</td>
</tr>
<tr>
<td>Level 3</td>
<td>FR12</td>
<td>Make y tops with</td>
<td>DP12</td>
<td>Machines for</td>
</tr>
</tbody>
</table>
drilling pattern TR1022 according to specs.
processing tops with drill head

<table>
<thead>
<tr>
<th>Level 3</th>
<th>FR111</th>
<th>Make 4x rails with dowels according to specs.</th>
<th>DP111</th>
<th>Machines for processing rails with dowel</th>
</tr>
</thead>
</table>

... ... ... ... ... ...

Take the functional module (k), FRs and DPs of the level 3, as a computation example, then the design matrix between the functional domain (FR111~FR113) and the physical domain (DP111~DP113) is constructed as follows.

\[
\begin{bmatrix}
FR_{111} \\
FR_{112} \\
FR_{113}
\end{bmatrix}
= \begin{bmatrix}
A_1 & A_2 & A_3 \\
A_21 & A_22 & A_23 \\
A_31 & A_32 & A_33
\end{bmatrix}
\begin{bmatrix}
DP_{111} \\
DP_{112} \\
DP_{113}
\end{bmatrix}
\]

Step 1. Construction of fuzzy evaluation matrix
There are three experts to participate in the evaluation.

Three linguistic valuation matrices are obtained as follows.

\[
\begin{bmatrix}
E & N & N \\
L & E & N \\
N & M & E
\end{bmatrix}
\begin{bmatrix}
E & N & N \\
L & E & N \\
N & M & E
\end{bmatrix}

Based on the Fig.2, transform these linguistic valuation matrices into the fuzzy evaluation matrices expressed by triangular fuzzy number, the final results are shown below.

\[
\begin{bmatrix}
A_1 \\
A_2 \\
A_3
\end{bmatrix}
= \begin{bmatrix}
(1,1,1) & (0,0,0) & (0,0,0) \\
(0,1,0.3,0.5) & (1,1,1) & (0,0,0) \\
(0,0,0) & (0.3,0.5,0.7) & (1,1,1)
\end{bmatrix}
\begin{bmatrix}
DP_{111} \\
DP_{112} \\
DP_{113}
\end{bmatrix}
\]

\[
\begin{bmatrix}
A_1 \\
A_2 \\
A_3
\end{bmatrix}
= \begin{bmatrix}
(1,1,1) \\
(0,0,0) \\
(0,0,0)
\end{bmatrix}
\begin{bmatrix}
(1,1,1) & (0,0,0) \\
(0.3,0.5,0.7) & (1,1,1) \\
(0,0,0)
\end{bmatrix}
\begin{bmatrix}
DP_{111} \\
DP_{112} \\
DP_{113}
\end{bmatrix}
\]

\[
\begin{bmatrix}
A_1 \\
A_2 \\
A_3
\end{bmatrix}
= \begin{bmatrix}
(1,1,1) & (0,0,0) & (0,0,0) \\
(0,1,0.3,0.5) & (1,1,1) & (0,0,0) \\
(0,0,0) & (0,1,0.3,0.5) & (1,1,1)
\end{bmatrix}
\begin{bmatrix}
DP_{111} \\
DP_{112} \\
DP_{113}
\end{bmatrix}
\]

Step 2. Construction of fuzzy comprehensive evaluation matrix
The number of experts is 3, so prior weight of each expert is \( \omega_i = \frac{1}{3} \).

Based on the Eq.9, the posterior weight of each expert can be obtained.

\[
\omega_i = \left[ \begin{array}{ccc}
0.333 & 0.333 & 0.333 \\
0.346 & 0.333 & 0.333 \\
0.333 & 0.346 & 0.333
\end{array} \right]
\]

\[
\omega_i = \left[ \begin{array}{ccc}
0.333 & 0.333 & 0.333 \\
0.307 & 0.333 & 0.333 \\
0.312 & 0.346 & 0.333
\end{array} \right]
\]

\[
\omega_i = \left[ \begin{array}{ccc}
0.333 & 0.333 & 0.321 \\
0.346 & 0.333 & 0.333 \\
0.354 & 0.307 & 0.333
\end{array} \right]
\]

Set the coefficient as \( \gamma = 0.3 \). The weight of each expert is calculated by Eq.10, and the results are shown as follows:

\[
\omega_i = \left[ \begin{array}{ccc}
0.333 & 0.333 & 0.333 \\
0.342 & 0.333 & 0.333 \\
0.333 & 0.342 & 0.333
\end{array} \right]
\]

\[
\omega_i = \left[ \begin{array}{ccc}
0.333 & 0.333 & 0.333 \\
0.315 & 0.333 & 0.333 \\
0.318 & 0.342 & 0.333
\end{array} \right]
\]

\[
\omega_i = \left[ \begin{array}{ccc}
0.333 & 0.333 & 0.325 \\
0.342 & 0.333 & 0.333 \\
0.348 & 0.315 & 0.333
\end{array} \right]
\]

Finally, the comprehensive evaluation matrix can be obtained by Eq.11.

\[
\lambda = \left[ \begin{array}{ccc}
0.163 & 0.363 & 0.536 \\
0.023 & 0.437 & 0.636 \\
0.111 & 0.111 & 0.111
\end{array} \right]
\]

Step 3. Computation of fuzzy coupling degree
According to the fuzzy comprehensive evaluation matrix \( \lambda \), the fuzzy coupling degree can be calculated. Due to the triangular fuzzy number, \( (0,0.033,0.065) \) is small enough to be ignored in this case, that is, \( \overline{\zeta} = 0.3 \). Based on the Eq.13, the value of the parameter \( c \) can be calculated as \( \overline{\zeta} = (0.144,0.310,0.437) \). Removing fuzziness and the coupling degree can be expressed as:

\[
E(c_i) = \frac{(a_i + 2a_{i\gamma} + a_{i\gamma\gamma})}{4}
\]

\[
= \frac{0.144 + 2 \times 0.31 + 0.437}{4}
\]

\[
= 0.300
\]

Step 4. Computation of fuzzy information content
Based on the test data for each component, the inherent reliability of each type of components is as follows:

\( R_{111} = 0.990; R_{112} = 0.942; R_{113} = 0.975 \)

Based on the Eq.15, the fuzzy information content can be calculated as:

\[
I_k = -\left(1 + E(c_i)\right) \log_2 \left( \prod_{i=1}^{k} R_{11i} \right)
\]

\[
= 0.054
\]

The computation result indicates the information content contained in this set of function design. Further, based on the information content, the designers can determine whether the set of functional requirements and the selection of design parameters meet the requirements. At the same time, it provides scientific guidance for reliability-oriented optimization design and verification.

5. Conclusions
In this paper, from the perspective of reliability-oriented complexity analysis of manufacturing systems in design stage, a novel approach to complexity analysis is proposed by introducing the triangular fuzzy number into AD. The approach focuses on the fuzziness and uncertainty of...
information and helps to build fuzzy evaluation matrices. Further, quantify the coupling degree and uncertainty in the form of information content. The proposed approach provides a way for reliability-oriented complexity analysis of manufacturing systems. In addition, it also should be an effective tool to guide the designers to carry out the reliability optimization design and verification.

However, this study is located at the design stage. When quantifying the uncertainty, only the inherent reliability of components is considered, and the dynamic change of manufacturing process is ignored. In order to improve the applicability, the study of dynamic complexity analysis of manufacturing systems which considering the actual operation is planned.

Acknowledgements

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References