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Finding the minimum information content in 2-FR, 2-DP coupled designs

João Fradinho[a,*], António Mourão[a], António Gabriel-Santos[a] & António Gonçalves-Coelho[a]

[a] *UNIDEMI & DEMI, Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, Campus de Caparica, Caparica 2829-516, Portugal*

* Corresponding author. Tel: +351-212-948-567; fax +351-212-948-531. *E-mail address*: jmf@fct.unl.pt

Abstract

Axiomatic Design classifies designs into three basic types: uncoupled, decoupled and coupled. The first type encompasses the ideal designs, where independence is always ensured, the second includes the designs where independence can be achieved using the right sequence to fine-tune the design parameters as to satisfy the given set of functional requirements, while the last comprises designs for which independence can never be achieved. Usually, coupled designs are avoided and designers are encouraged to redesign their solutions until an uncoupled or a decoupled one is achieved. Nevertheless, coupled solutions are often hard to avoid. This paper discusses this issue and uses a simple graphical example on how to adjust either the functional requirements or the design parameters of a 2-FR, 2-DP coupled design that is regarded as being uncoupled, as to attain the minimum information content.

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Keywords: coupled designs; information content; graphical method

1. Introduction

Axiomatic Design (AD) was born in 1978 at the M.I.T. [1] and became a well-known theory in 1990, after Nam P. Suh published his first book on the subject [2]. AD provides a powerful general-purpose framework for engineering design that conforms to the methods of modern science and that can be useful for researchers, instructors, students and practitioners.

AD envisions three basic types of designs that are known as uncoupled, decoupled and coupled [2]. Uncoupled designs are the ones for which the independence of the functional requirements (FR) is always assured. Decoupled designs are the ones in which independence is attained by using the right sequence to adjust the design parameters (DP) as to satisfy the given set of functional requirements. At last, coupled designs are the ones for which independence is never achieved.

Thus, coupled designs should be reworked until an uncoupled or a decoupled design could be found, which many times is hard to attain. Hence, it is right to find a suitable way to deal with coupled designs that one could not decouple.

This paper discusses this issue and uses a simple 2D geometric shape as a graphical example on how to adjust the design parameters of a 2-FR, 2-DP coupled design as to attain the smallest information content.

Nomenclature

FR_i	i^{th} functional requirement
DP_i	i^{th} design parameter
ΔFR_i	half design range of FR_i
ΔDP_i	half design range of DP_i
δFR_i	random variation of FR_i
δDP_i	random variation of DP_i
$\left(FR_1^*;FR_2^*\right)$	co-ordinates of the central point in the FR space
$\left(DP_1^*;DP_2^*\right)$	co-ordinates of the central point in the DP space

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2. The basics of Axiomatic Design

In the viewpoint of AD, every design can be depicted in the Customer, the Functional, the Physical and the Process domains, as shown in Fig. 1 [2].

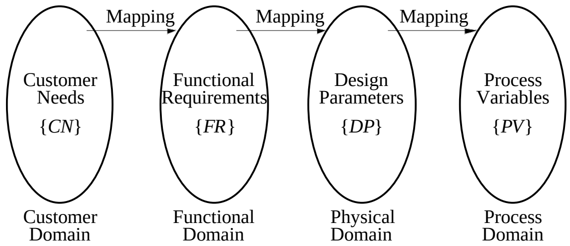

Fig. 1. The design domains

The entire description of a single-level design is attained by mapping from left to right across the design domains. Each mapping is represented by a design equation. As for example, mapping between the functional and the physical domains is denoted by the equation

$$\{FR\} = [A]\{DP\},$$
$$A_{ij} = \frac{\partial FR_i}{\partial DP_j}, i = 1..,m, j = 1,..,n \quad (1)$$

where $\{FR\}$ is the vector of functional requirements, $[A]$ is the design matrix and $\{DP\}$ is the vector of design parameters. The total numbers of FRs and DPs are expressed by m and n, respectively.

In the so-called “large designs” or “large systems” (*i.e.*, systems with many FRs that can be arranged with different levels of detail [3]), the mapping is deployed from left to right and from top to bottom, in a zigzag path that is unique to AD [2] and that discloses the design’s structure or architecture.

The Axiomatic Design reasoning is grounded on two self-ruling axioms that can be stated as follows [2]:

- The Independence Axiom – Maintain the independence of FRs. *Alternative statement*: In an acceptable design, the DPs and the FRs are related in such a way that specific DP can be adjusted to satisfy its corresponding FR without affecting other functional requirements.
- The Minimum Information Axiom – Minimize the information content. *Alternative statement*: The best design is a functionally uncoupled design that has the minimum information content.

Usually, the accomplishment of the Independence Axiom is checked first and frequently the critical design decisions are made as soon as the first solution that satisfies the Independence Axiom is found, because of the theorem [4]:

- Theorem 4 (Ideal design): In an ideal design, the number of DPs is equal to the number of FRs and the FRs are always maintained independent of each other.

Hence, the design matrix of Eq. 1 should be squared ($n = m$), and the designs will be *uncoupled* if the matrices are diagonal, *decoupled* if the matrices are triangular, or *coupled* if the matrices are populated in any other manner. Uncoupled designs are the best because the Independence Axiom is always obeyed independently of the order of adjusting the DPs as to attain the FRs. Decoupled designs conform to the Independence Axiom if the DPs are adjusted according to the right order. At last, coupled designs never conform to the Independence Axiom.

Although not being in the scope of this paper, it is worth noticing that designs with non-squared design matrices are also acknowledged by AD, for which the following theorems hold [4]:

- Theorem 1 (Coupling due to insufficient number of DPs): When the number of DPs is less than the number of FRs, either a coupled design results or the FRs cannot be satisfied
- Theorem 2 (Decoupling of coupled designs): When a design is coupled because of a larger number of FRs than DPs (*i.e.*, $m > n$), it may be decoupled by the addition of new DPs so as to make the number of FRs and DPs equal to each other if a subset of the design matrix containing $n \times n$ elements constitutes a triangular matrix.
- Theorem 3 (Redundant designs): When there are more DPs than FRs, the design is a redundant design, which can be reduced to an uncoupled design or a decoupled design, or a coupled design.

Typically, the Minimum Information Axiom is used to compare multiple design solutions, *i.e.*, multiple sets of DPs that fulfil the same set of FRs.

The AD’s definition of information content is borrowed from the Shannon’s theory of communication [5] and the computation of a 1-FR design is explained in Fig. 2. System range represents the whole ability of the system, and design range is the working range that the designer is looking for [2].

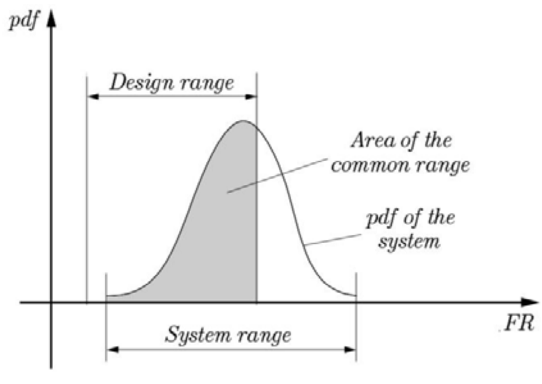

Fig. 2. The system range, the design range and the common range

Let us consider a 1-FR system which probability density function (*pdf*) is depicted in Fig. 2. The probability density function represents the probability of success of the system, so that the whole area of the system range is such as

$$\int_{-\infty}^{+\infty} p\,df(FR)\,dFR = 1 \qquad (2)$$

Fig. 2 also depicts the design range and the probability of the design fulfilling the FR is given by

$$p = \frac{\text{Area of the common range}}{\text{Area of the system range}} \qquad (3)$$

The information content of the design, I, expressed in *bit* is computed trough

$$I = \log_2 \frac{1}{p} = -\log_2 p \qquad (4)$$

The shape and the limits of the probability density function depend on the design under study. Most of the times, the shape of the *pdf* is a symmetrical or a skewed bell, but any other shape can occur, including the one that corresponds to a uniform distribution.

In the case of an *m*-FR design, the probability of success is

$$p_T = \prod_{i=1}^{m} p_i \qquad (5)$$

and the information content is therefore

$$I_T = \sum_{i=1}^{m} I_i \qquad (6)$$

The computation of the information content is simple for uncoupled designs because the probability of fulfilling each FR does not depend on the remaining FRs. However, this is not true in the case of decoupled or of coupled designs, for which the probability is conditional.

Suh [6] and Park [7] showed that a graphical method could be used to compute the information content of 2-FR, 2-DP decoupled designs when the probability density functions of both FRs have a uniform distribution. Yet, Park [7] noticed that usually the information content of coupled designs is not computed because they violate the Independence Axiom.

In fact, in the case of a 2-FR, 2-DP coupled design, one has

$$p\left(FR_1 \mid FR_2\right) p\left(FR_2 \mid FR_1\right) \neq p\left(FR_2 \mid FR_1\right) p\left(FR_1 \mid FR_2\right) \qquad (7)$$

which means that the computed value of the information content is path-dependent, as per AD's theorem 7 [2]. Therefore, the question is: can we find a way to regard a coupled design as being a decoupled design or, even better, as an uncoupled design?

3. The information content of a 2-FR, 2-DP coupled design

The expanded form of the equation of a 2-FR, 2-DP design is obtained from Eq. 1 [6, 7]

$$\begin{Bmatrix} FR_1 \\ FR_2 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} DP_1 \\ DP_2 \end{Bmatrix} \qquad (8)$$

In the finite differences form, Eq. 8 becomes

$$\begin{Bmatrix} \Delta FR_1 \\ \Delta FR_2 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} \Delta DP_1 \\ \Delta DP_2 \end{Bmatrix} \qquad (9)$$

According to Suh [6], the computation of the information content should be based on the intersection of the design range with the probability density functions. Fig. 3 depicts an isogram of a 2-FR, 2-DP coupled design in the functional space, considering that $A_{ij} > 0$.

ΔFR_i is related to the amplitude of the design ranges in the neighbourhood of point $\left(FR_1^*, FR_2^*\right)$, while δFR_i is the random variation of ΔFR_i.

The light grey parallelogram [ABCD] of Fig. 3 represents the system range, while the intersection of the system range with the design range (rectangle [EFGH]) is the common range (*i.e.*, the octagon [AJFKCLHI]) [8].

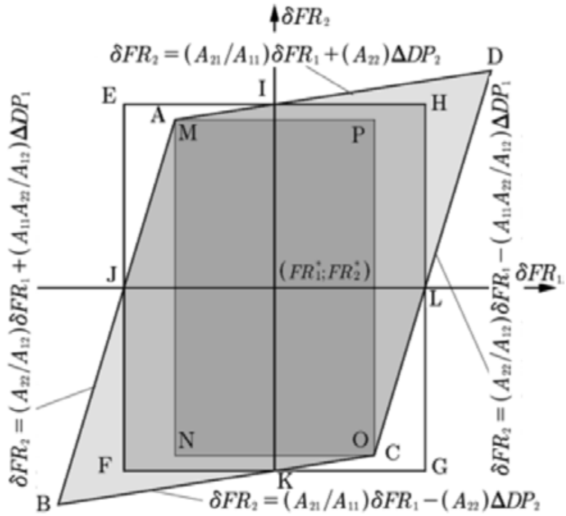

Fig. 3. The isogram of a 2-FR design in the functional space

The equations of the straight lines that form the system range are represented in Fig. 3, as shown by Suh [6] and Park [7], which co-ordinates of the most relevant points are:

$$A = \begin{pmatrix} \dfrac{A_{11}A_{22}}{A_{11}A_{22} - A_{12}A_{21}}\left(A_{12}\Delta DP_2 - A_{11}\Delta DP_1\right); \\ \dfrac{A_{11}A_{22}}{A_{11}A_{22} - A_{12}A_{21}}\left(A_{22}\Delta DP_2 - A_{21}\Delta DP_1\right) \end{pmatrix} \qquad (10)$$

$$B = \begin{pmatrix} \dfrac{-A_{22}\Delta DP_2 - \dfrac{A_{11}A_{22}}{A_{12}}\Delta DP_1}{\dfrac{A_{22}}{A_{12}} - \dfrac{A_{21}}{A_{11}}}; \\ \dfrac{-A_{22}^2\Delta DP_2 - \dfrac{A_{11}A_{22}^2}{A_{12}}\Delta DP_1}{A_{22} - \dfrac{A_{12}A_{21}}{A_{11}}} + \dfrac{A_{11}A_{22}}{A_{12}}\Delta DP_1 \end{pmatrix} \tag{11}$$

$$C = \begin{pmatrix} \dfrac{A_{11}A_{22}}{A_{11}A_{22} - A_{12}A_{21}}\left(-A_{12}\Delta DP_2 + A_{11}\Delta DP_1\right); \\ \dfrac{A_{11}A_{22}}{A_{11}A_{22} - A_{12}A_{21}}\left(-A_{22}\Delta DP_2 + A_{21}\Delta DP_1\right) \end{pmatrix} \tag{12}$$

$$D = \begin{pmatrix} \dfrac{A_{22}\Delta DP_2 + \dfrac{A_{11}A_{22}}{A_{12}}\Delta DP_1}{\dfrac{A_{22}}{A_{12}} - \dfrac{A_{21}}{A_{11}}}; \\ \dfrac{A_{22}^2\Delta DP_2 + \dfrac{A_{11}A_{22}^2}{A_{12}}\Delta DP_1}{A_{22} - \dfrac{A_{12}A_{21}}{A_{11}}} - \dfrac{A_{11}A_{22}}{A_{12}}\Delta DP_1 \end{pmatrix} \tag{13}$$

$$M \equiv A \tag{14}$$

$$N = \begin{pmatrix} \dfrac{A_{11}A_{22}}{A_{11}A_{22} - A_{12}A_{21}}\left(A_{12}\Delta DP_2 - A_{11}\Delta DP_1\right); \\ \dfrac{A_{11}A_{22}}{A_{11}A_{22} - A_{12}A_{21}}\left(-A_{22}\Delta DP_2 + A_{21}\Delta DP_1\right) \end{pmatrix} \tag{15}$$

$$O \equiv C \tag{16}$$

$$P = \begin{pmatrix} \dfrac{A_{11}A_{22}}{A_{11}A_{22} - A_{12}A_{21}}\left(-A_{12}\Delta DP_2 + A_{11}\Delta DP_1\right); \\ \dfrac{A_{11}A_{22}}{A_{11}A_{22} - A_{12}A_{21}}\left(A_{22}\Delta DP_2 - A_{21}\Delta DP_1\right) \end{pmatrix} \tag{17}$$

In this case, if the probabilities of fulfilling both FRs are uniform, then the information content, I_c, is given by

$$I_c = \log_2\left(\frac{\text{Area of parallelogram }[\text{ABCD}]}{\text{Area of octagon }[\text{AJFKCLHI}]}\right) \tag{18}$$

The dark grey rectangle [MNOP] depicts the region of the system range where both FRs are simultaneously fulfilled. This means that one can consider that the design is uncoupled when the design range coincides with the rectangle [MNOP].

Notice that rectangle [MNOP] is the largest rectangle that can be defined inside the system range. Therefore, if the design range coincides with the rectangle [MNOP], then the design can be seen as being uncoupled and the information content, I_u, will be

$$I_u = \log_2\left(\frac{\text{Area of parallelogram }[\text{ABCD}]}{\text{Area of rectangle }[\text{MNOP}]}\right) \tag{19}$$

One can see that $I_c < I_u$. However, I_c relates to a case where the simultaneous fulfilment of the FRs is not guaranteed, while the case related to I_u always ensures the fulfilment of both FRs.

Park [7] considered the standpoint on a 2-FR, 2-DP decoupled design depicted in the physical space. By extension, the case of a coupled design is shown in Fig. 4.

The physical space outlook is very useful when dealing with numerical simulation or with experimental tests.

In fact, if $A_{11}A_{22} > A_{12}A_{21}$, the inversion of Eq. 9 yields to

$$\begin{Bmatrix} \Delta DP_1 \\ \Delta DP_2 \end{Bmatrix} = \frac{1}{A_{11}A_{22} - A_{12}A_{21}}\begin{bmatrix} A_{22} & -A_{12} \\ -A_{21} & A_{11} \end{bmatrix}\begin{Bmatrix} \Delta FR_1 \\ \Delta FR_2 \end{Bmatrix} \tag{20}$$

The comparison of Eq. 9 with Eq. 20 shows that Fig. 4 is a mirrored image of Fig. 3 at a different scale.

The dark grey rectangle [MNOP] of Fig. 4 depicts the limit values of the DPs that allow regarding the design as being uncoupled.

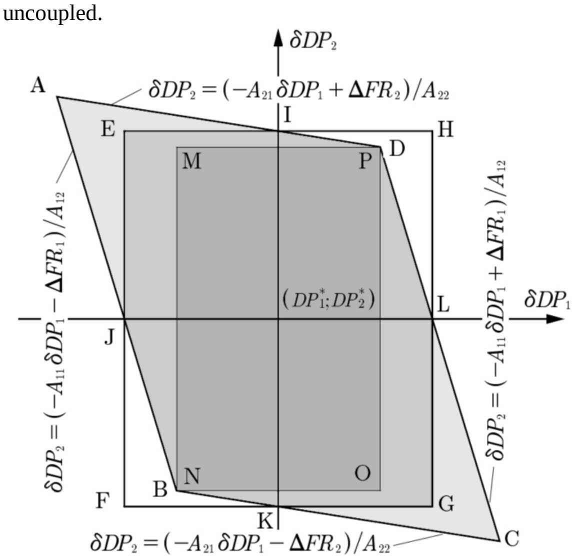

Fig. 4. The isogram of a 2-FR design in the physical space

The co-ordinates of the most important points in Fig. 4 are:

$$A = \begin{pmatrix} \dfrac{-A_{22}\Delta FR_1 - A_{12}\Delta FR_2}{A_{11}A_{22} - A_{12}A_{21}}; \\ \dfrac{A_{21}A_{22}\Delta FR_1 + A_{12}A_{21}\Delta FR_2}{A_{11}A_{22}^2 - A_{12}A_{21}A_{22}} + \dfrac{\Delta FR_2}{A_{22}} \end{pmatrix} \tag{21}$$

$$B = \begin{pmatrix} \dfrac{A_{22}\Delta FR_1 - A_{12}\Delta FR_2}{A_{12}A_{21} - A_{11}A_{22}}; \\ \dfrac{-A_{11}A_{22}\Delta FR_1 + A_{11}A_{12}\Delta FR_2}{A_{12}^2 A_{21} - A_{11}A_{12}A_{22}} - \dfrac{\Delta FR_1}{A_{12}} \end{pmatrix} \quad (22)$$

$$C = \begin{pmatrix} \dfrac{-A_{22}\Delta FR_1 - A_{12}\Delta FR_2}{A_{12}A_{21} - A_{11}A_{22}}; \\ \dfrac{A_{21}A_{22}\Delta FR_1 + A_{12}A_{21}\Delta FR_2}{A_{12}A_{21}A_{22} - A_{11}A_{22}^2} + \dfrac{\Delta FR_2}{A_{22}} \end{pmatrix} \quad (23)$$

$$D = \begin{pmatrix} \dfrac{A_{22}\Delta FR_1 - A_{12}\Delta FR_2}{A_{11}A_{22} - A_{12}A_{21}}; \\ \dfrac{-A_{11}A_{22}\Delta FR_1 + A_{11}A_{12}\Delta FR_2}{A_{11}A_{12}A_{22} - A_{12}^2 A_{21}} + \dfrac{\Delta FR_1}{A_{12}} \end{pmatrix} \quad (24)$$

$$M = \begin{pmatrix} \dfrac{A_{22}\Delta FR_1 - A_{12}\Delta FR_2}{A_{12}A_{21} - A_{11}A_{22}}; \\ \dfrac{-A_{11}A_{22}\Delta FR_1 + A_{11}A_{12}\Delta FR_2}{A_{11}A_{12}A_{22} - A_{12}^2 A_{21}} + \dfrac{\Delta FR_1}{A_{12}} \end{pmatrix} \quad (25)$$

$$N \equiv B \quad (26)$$

$$O = \begin{pmatrix} \dfrac{A_{22}\Delta FR_1 - A_{12}\Delta FR_2}{A_{11}A_{22} - A_{12}A_{21}}; \\ \dfrac{-A_{11}A_{22}\Delta FR_1 + A_{11}A_{12}\Delta FR_2}{A_{12}^2 A_{21} - A_{11}A_{12}A_{22}} - \dfrac{\Delta FR_1}{A_{12}} \end{pmatrix} \quad (27)$$

$$P \equiv D \quad (28)$$

It is worth to notice that the random variations δFR_i and δDP_i do not impact the co-ordinates of the points (Eq.s 10-17 and 21-28).

4. Discussion

Fig. 3 and Fig. 4 show the relationship between the functional and the physical domains that exists in a 2-FR, 2-DP coupled design. As one can see, it is possible to deal with the design as being uncoupled by limiting the design range to the rectangle [MNOP] in both Fig.s 3 and 4. Gonçalves-Coelho *et al.* noticed this fact without any remark for the special case of 3-FR, 2-DP coupled designs that are represented in the DP space [8]. In Fig. 4, which represents the design in the physical space, we can find the limits of DP_1 and DP_2 that allows considering the design as being uncoupled. Conversely, Fig. 3, which is related to the functional space, lets us finding the limits of FR_1 and FR_2 for the same design.

The big difference between Eq. 18 and Eq. 19 is that the former allows computing the minimum information content of 2-FR, 2-DP coupled designs (I_c), while the latter allows computing the minimum information content of the same designs considering them as being uncoupled (I_u).

The computation of the minimum information content for a rubber cork product was presented by Fradinho *et al.* [9]. This industrial example was treated in the physical space, considering it as being uncoupled.

5. Conclusions

Coupled designs are very common in the engineering field. Nevertheless, according to Axiomatic Design, they should be avoided. In fact, ideal designs are uncoupled, which represent instances where the FRs are always maintained independent of each other.

This paper deals with the 2-FR, 2-DP coupled designs with uniform probability densities. For this case, it was shown graphically how to determine the limits of the DP range (or conversely the limits of the FR range) that allow considering the design as being uncoupled. The resulting value of the information content, I_u, is greater than I_c, which is the information content of the real coupled design. However, I_u represents the minimum value of the information content that allows regarding the design as being uncoupled, because within these DP limits the achievement of both FRs is guaranteed.

In the future, we are planning to extend the graphical method for 3-FR, 3-DP designs using descriptive geometry techniques.

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